

Name:

**Problem 1** (2 points). Define a *group*  $(G, \cdot)$ .

**Problem 2** (4 points). Let  $G$  be a group, and  $g \in G$  be an element of order  $t$ . Show that if  $t = ab$  for some positive integers  $a, b$ , then the order of  $g^a$  is  $b$ .

**Problem 3** (4 points). Let  $G$  be a finite group of order  $n$  (i.e.,  $G$  has  $n$  distinct elements), and let  $g \in G$ . Show that the order of  $g$  is less than or equal to  $n$ .

**Problem 4** (Bonus). Let  $g$  be an element of a group  $G$ , and suppose that  $g$  has order  $n$ . Give a formula for the order of  $g^a$  in terms of  $a$  and  $n$ .