Name:

Problem 1 (4 points)**.** For each pair $(G, \cdot)$, where $\cdot$ is an operation on the set G, circle all the ones that are groups.

a) $(\mathbb{Z}, +)$.

b) $(\mathbb{Z}, \times)$.

c) $(\mathbb{N}, +)$.

d) $(\mathbb{R}_{>0}, \times)$.

e) $(\mathbb{R}_{\geqslant 0}, +)$.

f) $(\mathbb{Z}_{27}, +)$.

Problem 2 (3 points)**.** State the first isomorphism theorem for rings.

Problem 3 (3 points)**.** Consider the ring $R = \mathbb{Z}[x]$ and the ideal

$$I = \{p(x) \in R \text{ such that } 3 \mid p(0)\}.$$

Use the first isomorphism theorem for rings to prove that $R/I \cong \mathbb{Z}_3$.

Problem 4 (Bonus)**.** Prove the Third Isomorphism Theorem for rings:

If $I \subseteq J$ are ideals of the ring R, then the quotient ring $(R/I)/(J/I)$ is isomorphic to R/J.