Name:

Problem 1 (6 points)**.** Circle all the subsets that are ideals of the given ring.

a) The subset of $\mathbb{Z}$ of all even numbers.

b) The subset of $\mathbb{Z}$ of all odd numbers.

c) Matrices in $M_2(\mathbb{Z})$ with even entries.

d) All linear combinations of 6 and 9 in $\mathbb{Z}$.

e) $\{0\} \subseteq \mathbb{R}[x]$.

f) The subset of $\mathbb{Z}[x]$ all $p \in \mathbb{Z}[x]$ with $p(0) = 1$.

Problem 2 (4 points)**.** Consider the polynomials $p(x) = x^2 + 7x + 6$ and $q(x) = x^2 - 5x - 6$ in $\mathbb{Q}[x]$. Use the Euclidean algorithm to find their greatest common divisor.

Problem 3 (Bonus)**.** Let R be a commutative ring. An element $f \in R$ is called *nilpotent* if $f^n = 0$ for some $n \geqslant 1$. Show that the set of all nilpotent elements in R forms an ideal. Fun fact: this is called the nilradical of R.