Name: Solutions

Problem 1 (5 points)**.** Circle all the true statements. No justification required.

a) $\mathbb{Z}_7$ is a field.

b) $\mathbb{Z}_9$ is a domain. $3 \cdot 3 = 0$

c) Every domain is a field. $\mathbb{Z}$ domain, not a field

d) In a field, $ab = 0$ implies $a = 0$ or $b = 0$. field $\Rightarrow$ domain

e) Every subring of a field is a field. $\mathbb{Z} \subseteq \mathbb{Q}$ subring

f) Every subring of a domain is a domain.

Problem 2 (5 points)**.** Let R and S be rings, and $\varphi : R \to S$ be a ring homomorphism. Prove that the image of φ is a subring of S.

By definition of ring homomorphism,

$\varphi(a) + \varphi(b) = \varphi(a+b) \longrightarrow$ closed for $+$

$\varphi(a)\varphi(b) = \varphi(ab) \in$ image of $\varphi \longrightarrow$ closed for $\cdot$

$\varphi(1) = 1 \in$ image of φ

Also, $\varphi(0) = 0 \in$ image of φ, since

$\varphi(0) = \varphi(1-1) = \varphi(1) - \varphi(1) = 0$ so $0 \in$ image of φ

Moreover, for any element in the image of φ, say $\varphi(x)$,

$\varphi(x) + \varphi(-x) = \varphi(x-x) = \varphi(0) = 0$

so $\varphi(-x) = -\varphi(x)$ and the image of φ is closed for additive inverses.

$\therefore$ the image of φ is a subring of S.