Name:

Problem 0 (3 points)**.** Circle all the pairs $S \subseteq R$ where S is a subring of R.

(a) $\mathbb{Z} \subseteq \mathbb{Q}$. (b) $\mathbb{N} \subseteq \mathbb{Q}$. (c) $\varnothing \subseteq \mathbb{Z}$. (d) $\mathbb{R} \subseteq \mathbb{C}$.

Problem 1 (4 points)**.** Give an example of a *commutative* subring of the ring $M_2(\mathbb{R})$ of 2×2 matrices with real entries.

Problem 2 (3 points)**.** True or false? Justify your answer with a proof if it is true or a counterexample if it is false.

Let R be a ring and $a, b \in R$. If $ab = a$, then $ba = a$.

Problem 3 (Bonus)**.** An element r in a ring R is said to be an *idempotent* if $r^2 = r$. Prove that if R is a ring in which every element is an idempotent, then

- R is commutative, and
- $r + r = 0$ for every $r \in R$.