Name:

Problem 0 (2 points)**.** Which of the statements below *must* be true in any ring?

(a) Addition is associative.
(b) Multiplication is associative.
(c) Addition is commutative.
(d) Multiplication is commutative.
(e) Every element has an additive inverse.
(f) Every non-zero element has a multiplicative inverse.

Problem 1 (4 points)**.** Prove that if the equation $ax \equiv b \bmod n$ has a solution, then $(a, n)|b$.

Let $k \in \mathbb{Z}$ be a solution to the equation. Then for some q,

$ak - b = nq \iff b = ak - nq$.

Since $(a,n) \mid a$ and $(a,n) \mid n$, then

$(a,n) \mid ak - nq = b$.

Problem 2 (4 points)**.** True or false? Justify your answer with a proof if it is true or a counterexample if it is false.

$$\text{If } (a, n) = 1, \text{ then } [a] \text{ has a multiplicative inverse in } \mathbb{Z}_n.$$

True. If $(a,n) = 1$, there exist $u, v \in \mathbb{Z}$ such that

$ua + vn = 1$. Then

$ua \equiv 1 - vn \equiv 1 \pmod n$

and $[u]$ is a multiplicative inverse of $[a]$ in $\mathbb{Z}_n$.