Name:

Problem 0 (2 points). Which of the statements below *must* be true in any ring?

(a) Addition is associative.

(b) Multiplication is associative.

(c) Addition is commutative.

(d) Multiplication is commutative.

(e) Every element has an additive inverse.

(f) Every non-zero element has a multiplicative inverse.

Problem 1 (4 points). Prove that if the equation $ax \equiv b \bmod n$ has a solution, then $(a, n) | b$.

Problem 2 (4 points). True or false? Justify your answer with a proof if it is true or a counterexample if it is false.

$$\text{If } (a, n) = 1, \text{ then } [a] \text{ has a multiplicative inverse in } \mathbb{Z}_n.$$

Problem 3 (Bonus). If $a > b > 2$ are integers, then $2^b - 1$ does not divide $2^a + 1$.