Name:

Problem 0. Define the greatest common divisor of two integers a and b.

The greatest common divisor of a and b is the integer d s.t.

1) $d \mid a$ and $d \mid b$, and

2) For any integer c s.t. $c \mid a$ and $c \mid b$, $d \geq c$.

Problem 1. Use the Euclidean algorithm to write $(315, 525)$ as a linear combination of 315 and 525.

$$525 = 1 \cdot 315 + 210$$
$$315 = 1 \cdot 210 + 105$$
$$210 = 2 \cdot 105 + 0$$

The GCD is 105.

$$105 = 315 - 1 \cdot 210$$
$$= 315 - (525 - 315)$$
$$= (-1) \cdot 525 + 2 \cdot 315.$$

(other answers are possible)

Problem 2. True or false? Justify your answer with a proof if it is true or a counterexample if it is false.

For any positive integers a, b, and d, if $au + bv = d$ for some $u, v \in \mathbb{Z}$, then $(a, b) = d$.

False: For a counterexample, let $a = b = u = v = 1$ and $d = 2$:

$$\underset{a \cdot u + b \cdot v = d}{1 \cdot 1 + 1 \cdot 1 = 2}, \quad \text{but } (a, b) = (1, 1) = 1.$$