Name:

Problem 1 (3 points)**.** Define normal subgroup.

Problem 2 (4 points)**.** Prove that the kernel of a group homomorphism is a normal subgroup.

Problem 3 (3 points)**.** True or false: a group of order 15 can act on a set with 7 elements in such a way that there are exactly 2 orbits.

Problem 4 (Bonus)**.** Consider an action of a finite group G on a finite set X. Prove Burnside's Lemma[1]: the number of distinct orbits is equal to

$$\frac{1}{|G|}\sum_{g\in G}\big|\{x\in X \mid g\cdot x = x\}\big|.$$

[1] Fun fact: this was not proved by Burnside. Some call it *the lemma that is not Burnside's.*