Name:

Problem 1 (4 points)**.** Circle all the true statements, no justification necessary.

(a) D_7 contains an element of order 3.

(b) $\mathbb{Z}_4^\times \cong \mathbb{Z}_3^\times$.

(c) If G and H are groups of order 17, $G \cong H$.

(d) Faithful actions have no fixed points.

Problem 2 (3 points)**.** State Lagrange's Theorem.

Problem 3 (3 points)**.** True or false: given any group G of order 16 and any group H of order 24, there is no injective homomorphism $G \longrightarrow H$.

Problem 4 (Bonus)**.** Note that 113 is prime, so $\mathbb{Z}_{113}^{\times}$ is a cyclic group. Let $a \in \mathbb{Z}$. To check whether $[a]$ is a generator for $\mathbb{Z}_{113}^{\times}$ by using the definition naïvely, we would need to evaluate $a^n \bmod 113$ for all $n < 112$. By being clever, what is the fewest number of values of n for which we would need to check $a^n \not\equiv 1 \bmod 113$ to show that $[a]$ is a generator, and what are these values of n? Justify your answer.