Math 412
Midterm review problems

True or false. Justify!

1) In $\mathbb{Z}$, if $n = p_1 \cdots p_t = q_1 \cdots q_s$, for primes p_i, q_j, then $s = t$ and $p_1 = q_1, \ldots, p_s = q_s$. F
2) In general, the fastest way to find the gcd of two large integers is to factor them into primes. F
3) The equation $[a]_n x = [b]_n$ has a solution in $\mathbb{Z}_n$ if and only if $\gcd(a, n) = 1$. F
4) The system of equations $7|(x+3)$ and $11|(x-1)$ has a solution modulo 77. T
5) The system of equations $3|x$ and $6|(x-1)$ has a solution modulo 18. F
6) If $n|a$ and $m|a$, then $nm|a$. F
7) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z} \longrightarrow R$. T
8) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}$. F
9) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z}_n \longrightarrow R$. F
10) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}_n$. F
11) Every element in $\mathbb{Z}$ is a unit. F
12) The additive inverse of $[5]_{77}$ in $\mathbb{Z}_{77}$ is $[149]_{77}$. T
13) The multiplicative inverse of $[5]_{77}$ in $\mathbb{Z}_{77}$ is $[108]_{77}$. T
14) Every nonzero ring contains at least two ideals. T
15) Every domain is a field. F
16) Every field is a domain. T
17) The zero ring is a domain. F
18) There always exists a ring homomorphism between any two rings. F
19) Any commutative ring that has only two ideals is a field. T
20) The kernel of any ring homomorphism is an ideal. T
21) The kernel of any ring homomorphism is a subring. F
22) The image of any ring homomorphism is an ideal. F
23) The image of any ring homomorphism is a subring. T
24) If R is a commutative ring and $(g) = R$, then g is a unit. T
25) If R is a domain, then $R[x]$ is a domain. T
26) If F is a field, then $F[x]$ is a field. F
27) If $p(x) \in \mathbb{Z}_2[x]$ has degree 3, then $\mathbb{Z}_2[x]/(p(x))$ has 4 elements. F
28) If $p(x) \in F[x]$ for some field F is irreducible, then $\gcd(p(x), f(x))$ is 1 or p. T
29) If F is a field, the remainder of dividing $f(x)$ by $x - a$ is $f(a)$. T
30) Modern algebra is fun! T

31) The ring $\mathbb{Z}_n[x]$ is a domain. F
32) If f and g differ by a unit in $F[x]$, where F is a field, then $(f, g) = 1$. T
33) If $uf + vg = 4$ in $\mathbb{Q}[x]$, then $f + (g)$ is a unit in $\mathbb{Q}[x]/(g)$. T
34) In $R[x]$, the product of two monic polynomials can be zero. F
35) If F is a field, the map $F[x] \longrightarrow F$ sending each polynomial to its constant term is a ring homomorphism. T
36) $x^3 + 2$ is a unit in $\mathbb{Z}_5[x]/(x^4 - x^2)$. T
37) The quotient ring $\mathbb{R}[x]/(x^3 - x - 6)$ is a field. F
38) Every ideal is the kernel of some ring homomorphism. T
39) Any subring of a domain is a domain. T
40) Any subring of a field is a field. F
41) $2^3 \equiv 2^8 \mod 5$. F
42) Every integer is congruent to the sum of its digits modulo 11. F
43) An element of a commutative ring R cannot be both a unit and a zerodivisor. T
44) A subset of a ring that is also a ring is a subring. F
45) $\mathbb{Z}_n$ is a domain if and only if it is a field. T
46) If $ua + vb = n$ for some $a, b, u, v \in \mathbb{Z}$, then $(a, b) = n$. F
47) If $ua + vb = 1$ for some $a, b, u, v \in \mathbb{Z}$, then $(a, b) = 1$. T
48) Every element in $\mathbb{Z}_{11}$ is invertible. F
49) In $\mathbb{Z}_{77}$, $(a) = (b)$ if and only if $a = b$. F
50) Every ideal in $\mathbb{Z}_{123}$ is principal. T
51) In $\mathbb{Z}[x]$, $(a, b) = (\gcd(a, b))$. F
52) If R and S are domains, then $R \times S$ is a domain. F
53) In any ring R, $ab = 0$ implies $a = 0$ or $b = 0$. F
54) In any ring R, we can cancel addition. T
55) In any ring R, we can cancel multiplication. F
56) On the set of real numbers, $r \sim s$ if and only if $|r| = |s|$ defines an equivalence relation. T
57) If a is even and b is odd, (a, b) is even. F
58) If $a|b$ and $b|c$, then $a|c$. T
59) If I and J are ideals in a ring R, $I \cup J$ is an ideal in R. F
60) I'm an algebra whiz! T