Math 412
Midterm review problems

True or false. Justify!

1) In $\mathbb{Z}$, if $n = p_1 \cdots p_t = q_1 \cdots q_s$, for primes p_i, q_j, then $s = t$ and $p_1 = q_1, \ldots, p_s = q_s$.
2) In general, the fastest way to find the gcd of two large integers is to factor them into primes.
3) The equation $[a]_n x = [b]_n$ has a solution in $\mathbb{Z}_n$ if and only if gcd$(a, n) = 1$.
4) The system of equations $7|(x + 3)$ and $11|(x - 1)$ has a solution modulo 77.
5) The system of equations $3|x$ and $6|(x - 1)$ has a solution modulo 18.
6) If $n|a$ and $m|a$, then $nm|a$.
7) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z} \longrightarrow R$.
8) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}$.
9) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z}_n \longrightarrow R$.
10) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}_n$.
11) Every element in $\mathbb{Z}$ is a unit.
12) The additive inverse of $[5]_{77}$ in $\mathbb{Z}_{77}$ is $[149]_{77}$.
13) The multiplicative inverse of $[5]_{77}$ in $\mathbb{Z}_{77}$ is $[108]_{77}$.
14) Every nonzero ring contains at least two ideals.
15) Every domain is a field.
16) Every field is a domain.
17) The zero ring is a domain.
18) There always exists a ring homomorphism between any two rings.
19) Any commutative ring that has only two ideals is a field.
20) The kernel of any ring homomorphism is an ideal.
21) The kernel of any ring homomorphism is a subring.
22) The image of any ring homomorphism is an ideal.
23) The image of any ring homomorphism is a subring.
24) If R is a commutative ring and $(g) = R$, then g is a unit.
25) If R is a domain, then $R[x]$ is a domain.
26) If F is a field, then $F[x]$ is a field.
27) If $p(x) \in \mathbb{Z}_2[x]$ has degree 3, then $\mathbb{Z}_2[x]$ has 4 elements.
28) If $p(x)$ is irreducible, then gcd$(p(x), f(x))$ is 1 or p.
29) If F is a field, the remainder of dividing $f(x)$ by $x - a$ is $f(a)$.
30) Modern algebra is fun.

31) The ring $\mathbb{Z}_n[x]$ is a domain.
32) If f and g differ by a unit in $F[x]$, where F is a field, then $(f, g) = 1$.
33) If $uf + vg = 4$ in $\mathbb{Q}[x]$, then $f + (g)$ is a unit in $\mathbb{Q}[x]/(g)$.
34) In $R[x]$, the product of two monic polynomials can be zero.
35) If F is a field, the map $F[x] \longrightarrow F$ sending each polynomial to its constant term is a ring homomorphism.
36) $x^3 + 2$ is a unit in $\mathbb{Z}_5[x]/(x^4 - x^2)$.
37) The quotient ring $\mathbb{R}[x]/(x^3 - x - 6)$ is a field.
38) Every ideal is the kernel of some ring homomorphism.
39) Any subring of a domain is a domain.
40) Any subring of a field is a field.
41) $2^3 \equiv 2^8 \mod 5$.
42) Every integer is congruent to the sum of its digits modulo 11.
43) An element of a commutative ring R cannot be both a unit and a zerodivisor.
44) A subset of a ring that is also a ring is a subring.
45) $\mathbb{Z}_n$ is a domain if and only if it is a field.
46) If $ua + vb = n$ for some $a, b, u, v \in \mathbb{Z}$, then $(a, b) = n$.
47) If $ua + vb = 1$ for some $a, b, u, v \in \mathbb{Z}$, then $(a, b) = 1$.
48) Every element in $\mathbb{Z}_{11}$ is invertible.
49) In $\mathbb{Z}_{77}$, $(a) = (b)$ if and only if $a = b$.
50) Every ideal in $\mathbb{Z}_{123}$ is principal.
51) In $\mathbb{Z}[x]$, $(a, b) = (\text{gcd}(a, b))$.
52) If R and S are domains, then $R \times S$ is a domain.
53) In any ring R, $ab = 0$ implies $a = 0$ or $b = 0$.
54) In any ring R, we can cancel addition.
55) In any ring R, we can cancel multiplication.
56) On the set of real numbers, $r\tilde{s}$ if and only if $|r| = |s|$ defines an equivalence relation.
57) If a is even and b is odd, (a, b) is even.
58) If $a|b$ and $b|c$, then $a|c$.
59) If I and J are ideals in a ring R, $I \cup J$ is an ideal in R.
60) Modern algebra is fun.