LINEAR ALGEBRA REVIEW

Determine whether each statement is true when R is a field, is a PID, or is an arbitrary commutative ring (CR). Throughout, M, N are R-modules and A, B are matrices. We write $M \subseteq N$ to mean M is an R-submodule of N.

	Field	PID	CR
(1) Every M is free.			
(2) If M is torsionfree, then M is free.			
(3) If M is finitely generated and torsionfree, then M is free.			
(4) If M is finitely generated and free and $N \subseteq M$, then N is free.			
(5) If M is finitely generated and $N \subseteq M$, then N is fin. generated.			
(6) If M is finitely generated, $N \subseteq M$, and $N \cong M$, then $N = M$.			
(7) If M is finitely generated and $f : M \to M$ is injective, then f is an iso.			
(8) If M is finitely generated and $f : M \to M$ is surjective, then f is an iso.			
(9) If M is a free module and $N \subseteq M$, then N is free.			
(10) For any matrix A there are invertible P, Q s.t. PAQ is diagonal.			
(11) Any matrix A can be turned into a diagonal matrix with EROs and ECOs.			
(12) Any invertible matrix A can be turned into a diagonal matrix with EROs.			
(13) Any matrix A can be turned into a diagonal matrix with EROs.			
(14) An $n \times n$ matrix A is invertible if and only if $\det(A) \neq 0$.			
(15) An $n \times n$ matrix A is invertible if and only if the columns of A are LI.			
(16) An $n \times n$ matrix A is invertible if and only if $\exists B$ with $AB = I_n$.			
(17) An $n \times n$ matrix A is invertible if and only if $\exists B$ with $BA = I_n$.			
(18) If M is free and $S \subseteq M$ is LI, then S is a subset of a basis.			
(19) If M is free and $S \subseteq M$ generates M, then S contains a basis.			
(20) If $M \cong N$ are free then $\operatorname{rank}(M) = \operatorname{rank}(N)$.			
(21) If $M \twoheadrightarrow N$ are free then $\operatorname{rank}(M) \geq \operatorname{rank}(N)$.			
(22) If $M \subseteq N$ are free then $\operatorname{rank}(M) \leq \operatorname{rank}(N)$.			
(23) If $M \subseteq N$ and N can be generated by n elements then so can M.			
(24) If $M \subseteq N$ and N can be generated by n elements then so can N/M.			
(25) If $M \subseteq N$ and both M and N/M are free, then N is free.			

[0]Warning: There are a few tricky boxes here, but we can resolve almost all of them with techniques and examples we have considered in class.