PROBLEM SET #4

(1) Let $K = \mathbb{F}_3(s^2, st, t^2)$. Find a K vector space basis for the derivations from K to K, and for each basis element, evaluate it at the element s^3t.

(2) Compute the singular locus of the ring $\dfrac{\mathbb{F}_2(s,t)[x,y,z]}{(x^2+y^2z,\, y^2+sx^2+tz^2)}$.

(3) Let K be a field, and R be a K-algebra. Show that if R is finitely generated over K and reduced, then there is a maximal ideal $\mathfrak{m}$ of R such that $R_{\mathfrak{m}}$ is regular.

(4) Modify the proof of our example of a nonclosed singular locus to show that the ring $W^{-1}S$, where

$$S = K[x_{11}, x_{21}, x_{22}, x_{31}, x_{32}, x_{33}, \cdots] \qquad \text{and} \qquad W = S \smallsetminus \left(\bigcup_{j=1}^{\infty} (x_{j1}, \dots, x_{jj}) \right)$$

is a Noetherian ring of infinite Krull dimension.

(5) Let $(R, \mathfrak{m})$ be a Noetherian local ring. Show that every derivation $\partial : R \to R$ extends to a unique derivation $\hat{\partial} : \hat{R} \to \hat{R}$.

(6) Let $R = K[\![x_1 \dots, x_n]\!]$ be a power series ring over a field K.
  (a) Show that $\mathrm{Der}_{R|K}(R) = \sum_i R \frac{d}{dx_i}$.
  (b) Show that if K has characteristic $p > 0$, then for any R-module M, $\mathrm{Der}_{R|K}(M) = \sum_i M \frac{d}{dx_i}$.
  (c) What if K has characteristic 0?