

TRUE or FALSE. Justify.

- (1) There is a continuous function  $f : [2, 3] \rightarrow \mathbb{R}$  with range  $[3, 5]$ . **TRUE**
- (2) There is a continuous function  $f : [2, 3] \rightarrow \mathbb{R}$  with range  $(3, 5]$ . **FALSE**
- (3) There is a continuous function  $f : [2, 3] \rightarrow \mathbb{R}$  with range  $[3, 4] \cup [5, 6]$ . **FALSE**
- (4) Every continuous function is differentiable. **FALSE**
- (5) Every differentiable function is continuous. **TRUE**
- (6) If  $f'(r) < 0$ , then there is some  $\delta > 0$  such that  $f(x) > f(r)$  whenever  $r - \delta < x < r$ . **TRUE**
- (7) If  $f'(r) < 0$ , then there is some  $\delta > 0$  such that  $f(x) > f(r)$  whenever  $r < x < r + \delta$ . **FALSE**
- (8) The function  $f(x) = x^3 - 2x^2 + 5$  is increasing on the interval  $(1, 2)$ . **FALSE**
- (9) The function  $f(x) = x^3 - 2x^2 + 5$  is decreasing on the interval  $(1, 2)$ . **FALSE**
- (10) If  $f$  is differentiable on  $\mathbb{R}$ , and  $f'(x) > 0$  for all  $x \in \mathbb{R}$ , then  $f(a) < f(b)$  for all  $a < b$ . **TRUE**
- (11) The function  $f(x) = \begin{cases} x^3 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$  is differentiable on  $\mathbb{R}$ . **TRUE**
- (12) If  $f'(x) = 0$  for all  $x$  in a subset  $S \subseteq \mathbb{R}$ , then there is some  $c \in \mathbb{R}$  such that  $f(x) = c$  for all  $x \in S$ . **FALSE**
- (13) The function  $f(x) = \begin{cases} \frac{x}{2} + x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$  is differentiable on all of  $\mathbb{R}$ . **TRUE**
- (14) There is some  $\delta > 0$  such that  $\frac{x}{2} + x^2 \sin(\frac{1}{x}) > 0$  whenever  $0 < x < \delta$ . (Use derivatives to explain.) **TRUE**
- (15) There is some  $\delta > 0$  such that the function  $f(x) = \begin{cases} \frac{x}{2} + x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$  is increasing on  $(-\delta, \delta)$ . **FALSE**

- (16) Let  $\{a_n\}_{n=1}^{\infty}$ ,  $\{b_n\}_{n=1}^{\infty}$ , and  $\{c_n\}_{n=1}^{\infty}$  be sequences. The negation of the statement “If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  converge, then  $\{c_n\}_{n=1}^{\infty}$  converges” is “If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  converge, then  $\{c_n\}_{n=1}^{\infty}$  diverges”. **FALSE**
- (17) The commutative property/axiom of addition says that  $(x+y)+z = x+(y+z)$ . **FALSE**
- (18) Every nonempty set of real numbers that is bounded above has a maximum element. **FALSE**
- (19) If  $S$  is a set of real numbers and  $\sup(S) \in S$  then  $\sup(S)$  is the maximum element of  $S$ . **TRUE**
- (20) Every nonempty set of natural numbers that is bounded below has a minimum element. **TRUE**
- (21) Every nonempty set of integers that is bounded above has a maximum element. **TRUE**
- (22) The supremum of the set  $\{x \in \mathbb{Q} \mid x < \pi\}$  is  $\pi$ . **TRUE**
- (23) The supremum of the set  $\{x \in \mathbb{Q} \mid x > \pi\}$  is  $\pi$ . **FALSE**
- (24) If a sequence  $\{a_n\}_{n=1}^{\infty}$  converges to  $L$ , then there is some  $N \in \mathbb{R}$  such that for all natural numbers  $n > N$ ,  $a_n = L$ . **FALSE**
- (25) For every real number  $L$  there is a strictly decreasing sequence of rational numbers that converges to  $L$ . **TRUE**
- (26) A sequence of negative numbers can converge to a positive number. **FALSE**
- (27) Every decreasing sequence is convergent. **FALSE**
- (28) Every convergent sequence is bounded. **TRUE**
- (29) If  $\{a_n\}_{n=1}^{\infty}$  and  $\{b_n\}_{n=1}^{\infty}$  are convergent sequences, then  $\{3a_n^2 + a_nb_n - 6b_n\}_{n=1}^{\infty}$  is a convergent sequence. **TRUE**
- (30) If for every  $\varepsilon > 0$ , there is some  $N \in \mathbb{R}$  such that  $|a_n - a_{n+1}| < \varepsilon$  for all  $n > N$ , then  $\{a_n\}_{n=1}^{\infty}$  is a Cauchy sequence. **FALSE**
- (31) If  $\lim_{x \rightarrow -1} f(x) > 5$ , then there is some  $\delta > 0$  such that  $f(x) > 5$  whenever  $x \in (-1 - \delta, -1 + \delta) \setminus \{-1\}$ . **TRUE**
- (32) If  $\lim_{x \rightarrow -1} f(x) \geq 5$ , then there is some  $\delta > 0$  such that  $f(x) \geq 5$  whenever  $x \in (-1 - \delta, -1 + \delta) \setminus \{-1\}$ . **FALSE**

- (33) If  $\{a_n\}_{n=1}^{\infty}$  converges to 1 and  $\{b_n\}_{n=1}^{\infty}$  converges to  $-2$ , then  $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$  converges to  $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$ . **FALSE**
- (34) There is a sequence without any convergent subsequence. **TRUE**
- (35) Every convergent sequence is a bounded sequence. **TRUE**
- (36) If a sequence diverges to  $\infty$ , then it is not bounded above. **TRUE**
- (37) If a sequence is not bounded above, then it diverges to  $\infty$ . **FALSE**
- (38) If  $f$  and  $g$  are functions defined on  $\mathbb{R}$ , and  $f(x) = g(x)$  for all  $x \neq a$ , and  $f$  has a limit as  $x$  approaches  $a$ , then  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$ . **TRUE**
- (39) If  $\lim_{x \rightarrow 0} f(x) = 3$ , then the sequence  $\{a_n\}_{n=1}^{\infty}$  converges to 0, where  $a_n = f(1/n)/n$ . **TRUE**
- (40) If  $f$  and  $g$  are continuous on  $(-7, 7)$ , and  $g(4) = -1$ , then  $\lim_{x \rightarrow 4} (f \circ g)(x) = f(-1)$ . **TRUE**
- (41) The sequence  $\{a_n\}_{n=1}^{\infty}$  where  

$$a_n = n \cdot \sqrt{2} - \text{the largest integer that is less than } n \cdot \sqrt{2}$$
has a convergent subsequence. **TRUE**
- (42) If two different subsequences of  $\{a_n\}_{n=1}^{\infty}$  converge, then  $\{a_n\}_{n=1}^{\infty}$  converges. **FALSE**
- (43) The function  $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$  is continuous on  $\mathbb{R}$ . **FALSE**
- (44) If the domain of  $f$  is  $\mathbb{R}$ , then  $f$  is continuous at some point. **FALSE**
- (45) The function  $f(x) = |x^2 - |x^3 - 3||$  is continuous on  $\mathbb{R}$ . **TRUE**
- (46) If  $f(x)$  is continuous at  $x = a$  then  $\lim_{x \rightarrow a} f(x)$  exists. **TRUE**
- (47) If  $f$  is continuous on  $\mathbb{R}$  and  $a < b$ , and  $y > f(a) > f(b)$ , then there is no  $c \in [a, b]$  such that  $f(c) = y$ . **FALSE**
- (48) If  $f$  is continuous on  $\mathbb{R}$  and  $a < b$ , and  $f(a) > y > f(b)$ , then there is no  $c \in [a, b]$  such that  $f(c) = y$ . **FALSE**
- (49) There is a sequence  $\{a_n\}_{n=1}^{\infty}$  such that  $\{r \in \mathbb{R} \mid r \text{ is a limit of a subseq. of } a_n\} = [0, 7]$ . **TRUE**
- (50) There is a sequence  $\{a_n\}_{n=1}^{\infty}$  such that  $\{r \in \mathbb{R} \mid r \text{ is a limit of a subseq. of } a_n\} = (0, 7)$ . **FALSE**