Math 325-001 — Problem Set #4
Due: Monday, February 29 by midnight

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges.

(2) Prove that every bounded below, decreasing sequence converges.

(3) Assume $\{a_n\}_{n=1}^{\infty}$ converges to L and there is a real number T such that if $n \in \mathbb{N}$ and $n > T$, then $a_n \geq 0$. (In looser words: assume a_n is eventually nonnegative.) Prove $L \geq 0$.

(4) Suppose $\{a_n\}_{n=1}^{\infty}$ is a sequence that converges to 0 and that $a_n \geq 0$ for all n. Prove $\{\sqrt{a_n}\}_{n=1}^{\infty}$ converges to 0 too.

(5) Suppose $\{a_n\}_{n=1}^{\infty}$ converges to L and that $a_n \geq 0$ for all n. Prove that $\{\sqrt{a_n}\}_{n=1}^{\infty}$ converges to $\sqrt{L}$. [1]

(6) Prove the sequence $\{a_n\}_{n=1}^{\infty}$ where

$$a_n = \frac{7n^3 - 21n + 116}{6n^3 + 4n^2 + n + 1}$$

converges. (This includes finding the number that it converges to.) You should use our Theorem from class on computing limits, but be sure to carefully justify every step of your proof using this Theorem.

[1]Hint: Note that by problem (2), $L \geq 0$. For the case $L = 0$, you can just cite the previous problem. Finally, for the case $L > 0$, use $(\sqrt{a_n} - \sqrt{L})(\sqrt{a_n} + \sqrt{L}) = a_n - L$ to deduce that

$$|\sqrt{a_n} - \sqrt{L}| \leq \frac{|a_n - L|}{\sqrt{L}}.$$