Math 325-001 — Problem Set #1
Due: Friday, February 5 by midnight

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Prove $\sqrt{3}$ is irrational by mimicking the argument I used to prove $\sqrt{2}$ is irrational in lecture.

(2) For each of the following sets, which of the axioms of a field, listed in Theorem 1.1. of our text (page 5), do *not* hold if one replaces $\mathbb{Q}$ with the indicated set? Explain.
   (a) The set of nonnegative integers $\{0, 1, 2, 3, \dots\}$.
   (b) The set of nonnegative rational numbers $\{q \in \mathbb{Q} \mid q \geq 0\}$.
   (c) The set of all integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

(3) Prove the following "Cancellation of multiplication" property: If x, y and z are real numbers such that $xy = xz$ and $x \neq 0$, then $y = z$. Your proof should use nothing other than the axioms of the real numbers, just as I did in lecture to show Cancellation of Addition. (You will not need to use the completeness axiom).

(4) A real number that is not rational is called *irrational*.
   (a) Prove that if x is a real number and x^2 is irrational, then x is also irrational.
   (b) Prove that the following statement is false: "If x is an irrational number, then x^2 is also irrational." *Tip*: To show a statement is false you merely need to exhibit a specific instance when it fails to be true.

   above provided there exists a real number b such that $s \leq b$ for all $s \in S$.)

(5) Read Section 1.4 of our text. Then do #7 and #10 on pages 41–42. *Warning*: Be sure you are using the Second Edition of the text for these problems. If you are using the First Edition, contact me and I can give you the correct problem numbers.

(6) Do #9 and #11 on pages 41–42 of our text. *Warning*: Be sure you are using the Second Edition of the text for these problems. If you are using the First Edition, contact me and I can give you the correct problem numbers.