Problem Set 8

Due Thursday, October 30

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand. You cannot use any resources besides me, your classmates, and our course notes.

I will post the .tex code for these very spooky problems for you to use if you wish to type your homework. If you prefer not to type, please *write neatly.* As a matter of good proof writing style, please use complete sentences and correct grammar. You may use any result stated or proven in class or in a homework problem, provided you reference it appropriately by either stating the result or stating its name (e.g. the definition of ring or Lagrange's Theorem). Please do not refer to theorems by their number in the course notes, as that can change.

Problem 1. Prove that the converse to Lagrange's theorem is false: find a group G and a positive integer d such that d divides the order of G but G does not have any subgroups of order d.

Problem 2. Show that there are no simple groups of order 56.

Problem 3. Show that there are no simple groups of order $2^5 \cdot 7^3$.

Problem 4. Let G be a finite group of order pqr with $0 < p \leq q \leq r$ prime numbers[1]. Show that G is not simple.

[1] You should consider four cases:

1. $p < q < r$
2. $p = q < r$
3. $p < q = r$
4. $p = q = r$.