What to know for quizzes and exams

Definitions

(1) Rational number: We define a **rational number** to be a number expressible as a quotient of two integers: $\frac{m}{n}$ for $m, n \in \mathbb{Z}$ with $n \neq 0$.

(2) Contrapositive: The **contrapositive** of the statement "If P then Q" is the statement "If not Q then not P".

(3) Converse: The **converse** of the statement "If P then Q" is the statement "If Q then P".

(4) Irrational number: A real number is **irrational** if it is not rational.

(5) minimum / maximum: Let S be a set of real numbers. A **minimum** element of S is a real number x such that
   (a) $x \in S$, and
   (b) for all $y \in S$, $x \leq y$.

(6) Upper bound / lower bound: Let S be any subset of $\mathbb{R}$. A real number b is called an **upper bound** of S provided that for every $s \in S$, we have $s \leq b$.

(7) Bounded above / bounded below: A subset S of $\mathbb{R}$ is called **bounded above** if there exists at least one upper bound for S. That is, S is bounded above provided there is a real number b such that for all $s \in S$ we have $s \leq b$.

(8) Supremum: Suppose S is subset of $\mathbb{R}$ that is bounded above. A **supremum** of S is a number ℓ such that
   (a) ℓ is an upper bound of S (i.e., $s \leq \ell$ for all $s \in S$) and
   (b) if b is any upper bound of S, then $\ell \leq b$.

(9) Absolute value: For a real number x, the **absolute value** of x is $|x| := \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$

(10) (sequence) converges to L: Let $\{a_n\}_{n=1}^{\infty}$ be an arbitrary sequence and L a real number. We say $\{a_n\}_{n=1}^{\infty}$ **converges** to L if for every real number $\varepsilon > 0$, there is a real number N such that $|a_n - L| < \varepsilon$ for all natural numbers n such that $n > N$.

(11) (sequence is) convergent: We say that a sequence $\{a_n\}_{n=1}^{\infty}$ is **convergent** if there is a number L such that $\{a_n\}_{n=1}^{\infty}$ converges to L.

(12) (sequence is) divergent: We say that a sequence $\{a_n\}_{n=1}^{\infty}$ is **divergent** if there is no number L such that $\{a_n\}_{n=1}^{\infty}$ converges to L.

(13) increasing / decreasing sequence: We say that a sequence $\{a_n\}_{n=1}^{\infty}$ is **increasing** if for all $n \in \mathbb{N}$ we have $a_n \leq a_{n+1}$.

(14) strictly increasing / decreasing sequence: We say that a sequence $\{a_n\}_{n=1}^{\infty}$ is **strictly increasing** if for all $n \in \mathbb{N}$, $a_n < a_{n+1}$.

(15) monotone sequence: We say that a sequence $\{a_n\}_{n=1}^{\infty}$ is **monotone** if it is either decreasing or increasing.

(16) diverges to $+\infty$: A sequence **diverges to** $+\infty$ if for every real number M, there is some $N \in \mathbb{R}$ such that for every natural number $n > N$ we have $a_n > M$.

(17) diverges to $-\infty$: A sequence **diverges to** $-\infty$ if for every real number m, there is some $N \in \mathbb{R}$ such that for every natural number $n > N$ we have $a_n < m$.

(18) Subsequence: A **subsequence** of a given sequence $\{a_n\}_{n=1}^{\infty}$ is any sequence of the form $\{a_{n_k}\}_{k=1}^{\infty}$ where $\{n_k\}_{k=1}^{\infty}$ is any strictly increasing sequence of natural numbers.

(19) Limit of a function: Let f be a function and a, L be real numbers. We say that **the limit of** f **as** x **approaches** a **is** L if for any $\varepsilon > 0$ there exists $\delta > 0$ such that if $0 < |x - a| < \delta$, then x is in the domain of f and $|f(x) - L| < \varepsilon$.
(20) Continuous at a point: Let f be a function and a be a real number. We say f **is continuous at** a if for every $\varepsilon > 0$ there is a $\delta > 0$ such that if x is a real number such that $|x - a| < \delta$ then f is defined at x and $|f(x) - f(a)| < \varepsilon$.
(21) Continuous on an open interval: Let I be an open interval, and f be a function defined on I. We say that f is **continuous on the open interval** I if f is continuous at x for all $x \in I$.
(22) Continuous on a closed interval: Given a function $f(x)$ and real numbers $a < b$, we say f is **continuous on the closed interval** $[a, b]$ provided
   (a) f is continuous on the open interval (a, b),
   (b) for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $a \leq x < a + \delta$, then $f(x)$ is defined and $|f(x) - f(a)| < \varepsilon$, and
   (c) for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $b - \delta < x \leq b$, then $|f(x) - f(b)| < \varepsilon$.
(23) Differentiable: Let f be a function and r be a real number. We say f is **differentiable at** r is f is defined at r and the limit $\lim_{x \to r} \frac{f(x) - f(r)}{x - r}$ exists.
(24) Derivative (at a point): Let f be a function and r be a real number. We say that the **derivative of** f **at** r is the number $\lim_{x \to r} \frac{f(x) - f(r)}{x - r}$ provided this limit exists.
(25) Increasing/decreasing function: Let f be a function, and $S \subseteq \mathbb{R}$ be a set of real numbers contained in domain of f. We say that f is **increasing** on S if for any $a, b \in S$ with $a < b$ we have $f(a) \leq f(b)$.