PRACTICE ASSIGNMENT

This problem set is not to be turned in, but consists of practice problems on continuous functions.

(1) Using any suitable Theorems about continuous functions, show that the function
$$f(x) = \sqrt{|x^3 - x - 5|}$$
is continuous on $\mathbb{R}$.

(2) Show that the function with domain $\mathbb{R}$ given by the rule
$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q}, \text{ and} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$
is continuous at $x = 0$, but not at any other value of x.

DEFINITION 30.1: Given a function $f(x)$ and real numbers $a < b$, we say f is **continuous on the closed interval** $[a, b]$ provided

- for every $r \in (a, b)$, f is continuous at r in the sense defined already,
- for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $x \in [a, b]$ and $a \leq x < a + \delta$, then $|f(x) - f(a)| < \varepsilon$, and
- for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $x \in [a, b]$ and $b - \delta < x \leq b$, then $|f(x) - f(b)| < \varepsilon$.

(3) Let f be a function defined on the closed interval $[a, b]$. Show that f is continuous on the closed interval $[a, b]$ in the sense of our definition if and only if
- for every $r \in (a, b)$, f is continuous at r in the sense defined already,
- $\lim_{x \to a^+} f(x) = f(a)$, and
- $\lim_{x \to b^-} f(x) = f(b)$.

(4) Let f be a function continuous on $[a, b]$ and $r \in [a, b]$. Let $c \in \mathbb{R}$. Show that if $f(r) < c$, then there is some $\delta > 0$ such that for every $x \in [a, b]$ with $|x - r| < \delta$, we have $f(x) < c$.