ASSIGNMENT #7: DUE TUESDAY, NOVEMBER 5 AT MIDNIGHT

This problem set is to be turned in on Canvas. You may reference any result or problem from our worksheets or lectures, unless it is the fact to be proven! You are encouraged to work with others, but you should understand everything you write. Please consult the class website for acceptable/unacceptable resources for the problem sets.

(1) Using any theorems about limits and/or examples of limits from class and standard facts about the cosine function, compute

$$\lim_{x\to 0}\left(\frac{2x+3}{x^2+5}+x\cos\left(\frac{1}{x^5}\right)\right).$$

(2) Let $f(x)$ be the function with domain $\mathbb{R}$ given by the rule

$$f(x)=\begin{cases}1 & \text{if } x\in\mathbb{Z}\text{, and}\\ -1 & \text{if } x\notin\mathbb{Z}.\end{cases}$$

Prove that for any $a\in\mathbb{R}$, we have $\lim\limits_{x\to a} f(x)=-1$.

(3) Let $f(x)$ be the function with domain $\mathbb{R}$ given by the rule

$$f(x)=\begin{cases}x & \text{if } x\in\mathbb{Q}\text{, and}\\ 0 & \text{if } x\notin\mathbb{Q}.\end{cases}$$

(a) Prove that $\lim\limits_{x\to 0} f(x)=0$.

(b) Prove[1] that if $a\neq 0$, then $\lim\limits_{x\to a} f(x)$ does not exist.

DEFINITION: Let f be a function and $a\in\mathbb{R}$. We say that **the limit of $f(x)$ as x approaches a from the right is** L provided:

For every $\varepsilon>0$, there is some $\delta>0$ such that for all x satisfying $a<x<a+\delta$, we have that f is defined at x and also that $|f(x)-L|<\varepsilon$.

In this case, we write $\lim\limits_{x\to a^+} f(x)=L$. We define $\lim\limits_{x\to a^-} f(x)=L$ similarly (with $a-\delta<x<a$ in place of $a<x<a+\delta$).

(4) Use the definitions to show that $\lim\limits_{x\to 0}\sqrt{x}$ does not exist, but $\lim\limits_{x\to 0^+}\sqrt{x}=0$.

(5) Let f be a function and L be a real number. Prove that $\lim\limits_{x\to a} f(x)=L$ if and only if $\lim\limits_{x\to a^+} f(x)=L$ and $\lim_{x\to a^-} f(x)=L$.

[1] You can use the fact from an old worksheet that for any real number x, there is a sequence $\{r_n\}_{n=1}^{\infty}$ of rational numbers that converges to x, and there is another sequence $\{z_n\}_{n=1}^{\infty}$ of rirational numbers that converges to x.