

ASSIGNMENT #6: DUE TUESDAY, OCTOBER 29 AT MIDNIGHT

This problem set is to be turned in on Canvas. You may reference any result or problem from our worksheets or lectures, unless it is the fact to be proven! You are encouraged to work with others, but you should understand everything you write. Please consult the class website for acceptable/unacceptable resources for the problem sets.

- (1) For each of the following, give an explicit example as indicated; no proofs are necessary.
 - (a) A sequence that has a subsequence that converges to 1, another subsequence that converges to 2, and a third subsequence that converges to 3.
 - (b) A sequence that has one sequence that is monotone and converges to 0 and another subsequence that is monotone and diverges to $+\infty$.
 - (c) A sequence of natural numbers such that for every natural number n there is a subsequence converging to n .
 - (2) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence. Show that if $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$, then every subsequence of $\{a_n\}_{n=1}^{\infty}$ diverges to $+\infty$ as well.
 - (3) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence and L a real number. Show that $\{a_n\}_{n=1}^{\infty}$ converges to L if and only if both of the subsequences $\{a_{2k}\}_{k=1}^{\infty}$ and $\{a_{2k+1}\}_{k=1}^{\infty}$ converge to L .
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- (4) Using just the $\varepsilon - \delta$ definition of limit, show that

$$\lim_{x \rightarrow -1} \frac{x^2 - 6x - 7}{x + 1} = -8.$$

- (5) Using just the $\varepsilon - \delta$ definition of limit, show that¹ for any $a > 0$, $\lim_{x \rightarrow a} \sqrt{x} = \sqrt{a}$.

¹Hint: You may want to use that $|\sqrt{x} - \sqrt{a}| = \frac{|x - a|}{|\sqrt{x} + \sqrt{a}|} \leq \frac{|x - a|}{|\sqrt{a}|}$.