ASSIGNMENT #4: DUE THURSDAY, OCTOBER 3 AT MIDNIGHT

This problem set is to be turned in on Canvas. You may reference any result or problem from our worksheets or lectures, unless it is the fact to be proven! You are encouraged to work with others, but you should understand everything you write. Please consult the class website for acceptable/unacceptable resources for the problem sets.

(1) Prove that the sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ diverges.

(2) Assume that $\{a_n\}_{n=1}^{\infty}$ converges to zero, and that $a_n \geq 0$ for all natural numbers n. Show[1] that $\{\sqrt{a_n}\}_{n=1}^{\infty}$ converges to zero also.

(3) Prove that the sequence $\left\{\dfrac{-3n^2+4}{4n^2-n+3}\right\}_{n=1}^{\infty}$ converges to $\dfrac{-3}{4}$. You should use our Theorem on Limits and Algebra, but explain carefully each step how you apply the Theorem.

(4) Find, with justification, examples of sequences such that:
   (a) $\{a_n\}_{n=1}^{\infty}$ gets closer and closer to to 7 (meaning $\{|a_n - 7|\}_{n=1}^{\infty}$ is strictly decreasing) but $\{a_n\}_{n=1}^{\infty}$ does not converge to 7.
   (b) Every term of $\{b_n\}_{n-1}^{\infty}$ is within 0.0000001 of π (meaning for all $n \in \mathbb{N}$, we have $|b_n - \pi| < 0.0000001$), but $\{b_n\}_{n=1}^{\infty}$ does not converge to π.
   (c) The first 1000 terms of $\{c_n\}_{n=1}^{\infty}$ are all larger than 1000000, but $\{c_n\}_{n=1}^{\infty}$ converges to 0.

[1] Warning! It is *not* true that $\sqrt{r} \leq r$ for all nonnegative numbers r.