ASSIGNMENT #3: DUE THURSDAY, SEPTEMBER 26 AT MIDNIGHT

This problem set is to be turned in on Canvas. You may reference any result or problem from our worksheets or lectures, unless it is the fact to be proven! You are encouraged to work with others, but you should understand everything you write. Please consult the class website for acceptable/unacceptable resources for the problem sets.

(1) Let S be a set of real numbers and $T = \{x^2 \mid x \in S\}$.
   (a) Prove that if T is bounded above, then S is bounded above.
   (b) Is the converse to this statement true? Prove or disprove.

(2) Let r be any real number. Prove that the supremum of the set $S_r = \{q \in \mathbb{Q} \mid q < r\}$ is r.

(3) Use the definition of "converges" to show that the sequence $\left\{\frac{1}{\sqrt{n}}\right\}_{n=1}^{\infty}$ converges to 0.

(4) Guess a value that the sequence $\left\{\frac{3n^2}{7n^2+1}\right\}_{n=1}^{\infty}$ converges to and use the definition of "converges" to prove that your answer is correct.

(5) Let $\{b_n\}_{n=1}^{\infty}$ be a sequence, and suppose that b_n converges to 5.
   (a) Show that there is some number N such that $b_n \in (3, 7)$ for all natural numbers $n > N$.
   (b) Show that there is some number N such that $b_n < 5.01$ for all natural numbers $n > N$.
   (c) Prove or disprove: There is some number N such that $b_n = 5$ for all natural numbers $n > N$.