TRUE/FALSE. JUSTIFY AND/OR CORRECT.

(1) If (a, b) is a solution of the Pell's equation, then $a+b\sqrt{D}$ is a unit in $\mathbb{Z}[\sqrt{D}]$.

(2) If $a+b\sqrt{D}$ is a unit in $\mathbb{Z}[\sqrt{D}]$, then (a, b) is a solution of the Pell's equation.

(3) The norm function on $\mathbb{Z}[\sqrt{D}]$ satisfies the property $N(\alpha+\beta) = N(\alpha) + N(\beta)$.

(4) For any integer D, the equation $x^2 - Dy^2 = 1$ has infinitely many solutions.

(5) If D is a squarefree integer and m is any integer, then the equation $x^2 - Dy^2 = m$ has a solution.

(6) If D is a squarefree integer and m is any integer, then the equation $x^2 - Dy^2 = m$ has either infinitely many solutions or no solutions.

(7) The solutions to any Pell's equation forms a group.

(8) A real number has a finite continued fraction expansion if and only if it is rational.

(9) Every irrational number α can be approximated by infinitely many real numbers $\frac{p}{q}$ with $|\alpha - \frac{p}{q}| < \frac{1}{q^2}$.

(10) Every irrational number α can be approximated by infinitely many real numbers $\frac{p}{q}$ with $|\alpha - \frac{p}{q}| < \frac{1}{q^3}$.

(11) The solution set in $\mathbb{R}^2$ to any polynomial $y^2 = x^3 + ax + b$ forms a group.

(12) There exists an elliptic curve with exactly 9 rational points (not including ∞) that has a rational inflection point.

COMPUTATIONS

(1) Find the inverse of $[56]$ in $\mathbb{Z}_{89}$.

(2) Compute $\left(\frac{-15}{43}\right)$.

(3) Compute $\left(\frac{37}{113}\right)$.

(4) Find the continued fraction expansion of $\frac{1}{\sqrt{3}}$.

(5) Given that $\sqrt{2} = [1; 2, 2, 2, 2, \dots]$ and without using the decimal expansion of $\sqrt{2}$, determine which rational number with denominator less than 30 most closely approximates $\sqrt{2}$.

(6) Find the first three convergents for the continued fraction of $\sqrt[3]{2}$.

(7) Find the first three positive integer solutions of the equation $x^2 = 7y^2 + 1$.

(8) Give an expression for all solutions of the Pell's equation $x^2 - 12x^2 = 1$.

(9) Find the positive integer smallest solution of the equation $x^2 - 45y^2 = 1$.

(10) For the points $P = (2, 2)$, $Q = (0, 2)$ in the real elliptic curve $\overline{E}$ given by $y^2 = x^3 - 4x + 4$, compute $P \star Q$ and $P \star P$.

(11) Compute the order of the point $P = ([3], [0])$ in the elliptic curve $\overline{E}_5$ given by $y^2 = x^3 + x$ over $\mathbb{Z}_5$.

(12) Compute the order of the point $P = ([3], [1])$ in the elliptic curve $\overline{E}_5$ given by $y^2 = x^3 + x + [1]$ over $\mathbb{Z}_5$.