

# WHAT TO KNOW FOR QUIZZES AND EXAMS

## DEFINITIONS

- (1) Rational number
- (2) Contrapositive
- (3) Converse
- (4) Irrational number
- (5) minimum / maximum
- (6) Upper bound / lower bound
- (7) Bounded above / bounded below
- (8) Supremum / infimum
- (9) Absolute value
- (10) (sequence) converges to  $L$
- (11) (sequence) is convergent
- (12) (sequence) is divergent
- (13) increasing / decreasing sequence
- (14) strictly increasing / decreasing sequence
- (15) monotone sequence
- (16) diverges to  $+\infty$  or  $-\infty$
- (17) Subsequence
- (18) Limit of a function
- (19) Continuous at a point
- (20) Continuous on an open interval
- (21) Continuous on a closed interval
- (22) Differentiable
- (23) Derivative (at a point)
- (24) Derivative (function)
- (25) Increasing/decreasing function

## AXIOMS/THEOREMS

- (1) Well-ordering axiom
- (2) Completeness axiom
- (3) Theorem 5.3 (large natural numbers)
- (4) Archimedean principle
- (5) Density of rational numbers / irrational numbers
- (6) Triangle inequality
- (7) Theorem 10.2 (limits and algebra)
- (8) Squeeze Theorem
- (9) Monotone convergence theorem
- (10) Principle of induction
- (11) Theorem on convergence and subsequences
- (12) Cantor's Theorem
- (13) Bolzano-Weierstrass
- (14) Main corollary of Bolzano-Weierstrass
- (15) Theorem on limits and sequences
- (16) Theorem on limits of functions and algebra

- (17) Squeeze Theorem for functions
- (18) Theorem on continuity and limits
- (19) Theorem on continuity and algebra
- (20) Theorem on continuity and compositions
- (21) Intermediate Value Theorem
- (22) Boundedness Theorem
- (23) Extreme Value Theorem
- (24) Derivatives and algebra
- (25) Derivatives and minima/maxima
- (26) Mean Value Theorem
- (27) Increasing/decreasing functions and derivatives

#### KEY SKILLS

- (1) Proving “if-then” statements, “for every” statements, “there exists” statements, “is unique” statements
- (2) Proofs by contradiction
- (3) Finding the negation of a statement
- (4) Finding the contrapositive of a statement
- (5) Using examples to prove / disprove statements
- (6) Proving that a number is the supremum of a set
- (7) Proving that a sequence converges to some value using the definition
- (8) Algebra with limits of sequences: using these to determine if a sequence converges, and to what
- (9) Using squeeze theorem to show sequences converge
- (10) Relationship between boundedness, convergence, and monotonicity
- (11) Proofs by induction
- (12) Relationship between convergence/boundedness of sequences and convergence of sub-sequences
- (13) Using the  $\varepsilon - \delta$  definition to compute limits
- (14) Using algebra/squeeze theorem to compute limits
- (15) Using the  $\varepsilon - \delta$  definition to show continuity
- (16) Using algebra/compositions to show continuity
- (17) Applying the  $\varepsilon - \delta$  definitions of limits and continuity
- (18) Applying the Intermediate Value Theorem
- (19) Applying Boundedness and Extreme Value Theorems
- (20) Evaluating derivatives by definition
- (21) Evaluating derivatives by algebra
- (22) Using definition of derivative and mean value theorem to determine increasing / decreasing behavior of functions