True or False. Justify.

(1) Every bounded sequence is a convergent sequence.

(2) If a sequence has a divergent subsequence, then it diverges.

(3) The limit of $f(x) = \frac{x^2-2x+3}{x-7}$ as x approaches 3 is $-3/2$.

(4) The function $f(x) = \cos(1/x)$ has a limit as x approaches 0.

(5) If $\lim_{x\to -1} f(x)$ and $\lim_{x\to -1} g(x)$ both exist, then $\lim_{x\to -1} f(x)g(x)$ exists.

(6) If f is a function defined on $\mathbb{R}$, $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2, and $\lim_{x\to 0} f(x) = L$, then $L = 2$.

(7) If f is continuous at 2, $f(2) = 3$, and $\lim_{x\to 1} g(x) = 2$, then $\lim_{x\to 1}(f \circ g)(x) = 3$.

(8) If $\{a_n\}_{n=1}^{\infty}$ converges to 1 and $\{b_n\}_{n=1}^{\infty}$ converges to -2, then $\{a_{3n-1}b_n - b_{n^2}/4\}_{n=1}^{\infty}$ converges to $-5 = (3 \cdot 1 - 1)(-2) - (-2)^2/4$.

(9) For a given sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.

(10) The function $f(x) = \frac{x^2-2x+3}{x-7}$ is continuous on $(7, \infty)$.

(11) The function $f(x) = \sqrt{x^4 + 4x^2 + 5}$ is continuous on $\mathbb{R}$.

(12) If the domain of f is $\mathbb{R}$, then f is continuous at some point.

(13) If $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ does not exist, then $\lim_{x\to a} f(x) + g(x)$ does not exist.

(14) If f is continuous at a and $\lim_{x\to a} f(x) \geq 5$, then there is some $\delta > 0$ such that $f(x) \geq 5$ for all $x \in (a - \delta, a + \delta)$.

(15) There exists a sequence $\{a_n\}_{n=1}^{\infty}$ such that

$$\{r \in \mathbb{R} \mid \text{there is a subsequence of } \{a_n\}_{n=1}^{\infty} \text{ that converges to } r\} = [0, 3].$$

True or False. Justify.

(16) Every sequence has a bounded subsequence.

(17) There is a sequence without any monotone subsequence.

(18) The limit of $f(x) = \sqrt{4 - x^2}$ as x approaches 2 is 0.

(19) If $\lim_{x\to -1} f(x)/g(x) = 1$, then $\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x)$.

(20) If $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 1} g(x) = 2$, then $\lim_{x\to 1}(f \circ g)(x) = 3$.

(21) If $\lim_{x\to 0} f(x) = 2$, then the sequence $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2.

(22) If f is a function defined on $\mathbb{R}$ and $\{f(1/n)\}_{n=1}^{\infty}$ converges to 2, then $\lim_{x\to 0} f(x) = 2$.

(23) The sequence $a_n = \sqrt{\pi n - \lfloor \pi n \rfloor}$ has a convergent subsequence, where $\lfloor x \rfloor$ denotes the largest integer that is smaller than x.

(24) For a given convergent sequence, there are at most two real numbers that occur as limits of subsequences of the sequence.

(25) The function $f(x) = \frac{x^2-2x+3}{x-7}$ is continuous on $\mathbb{R}$.

(26) If $\lim_{x\to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$.

(27) If f is continuous at a, then there exists some $\delta > 0$ such that f is continuous on $(a - \delta, a + \delta)$.

(28) If $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ does not exist, then $\lim_{x\to a} f(x)g(x)$ does not exist.

(29) If f is continuous at a and $\lim_{x\to a} f(x) > 5$, then there is some $\delta > 0$ such that $f(x) > 5$ for all $x \in (a - \delta, a + \delta)$.

(30) There exists a sequence $\{a_n\}_{n=1}^{\infty}$ such that

$$\{r \in \mathbb{R} \mid \text{there is a subsequence of } \{a_n\}_{n=1}^{\infty} \text{ that converges to } r\} = (0, 3).$$