

(3) Prove or disprove: If $\{a_n\}_{n=1}^{\infty}$ diverges to $-\infty$ then $\{-a_n\}_{n=1}^{\infty}$ diverges to $+\infty$.

Proof:

Let $\{a_n\}_{n=1}^{\infty}$ diverge to $-\infty$.

To show that $\{-a_n\}_{n=1}^{\infty}$ diverges to $+\infty$,

let $M \in \mathbb{R}$. By def. of " $\{a_n\}_{n=1}^{\infty}$ diverges to $-\infty$ " applied with the real number $-M$, there exists some $N \in \mathbb{R}$ such that for $n > N$ we have $a_n < -M$.

For such an n , we thus have $-a_n > M$.

Thus, for the same N , if $n > N$, then $-a_n > M$. This shows that $\{-a_n\}_{n=1}^{\infty}$ diverges to $+\infty$.