Math 325-002 — Problem Set #8
Due: Thursday, November 3 by 7 pm, on Canvas

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

> To prove that $\lim_{x \to a} f(x) = L$ by the definition we:
> - Let $\epsilon > 0$.
> - Take $\delta =$ [some number computed from scratch work that makes $f(x)$ defined and $|f(x) - L| < \epsilon$ true for all x satisfying $0 < |x - a| < \delta$.]
> - Let x be a real number such that $0 < |x - a| < \delta$.
> - [Argument that $f(x)$ is defined and $|f(x) - L| < \epsilon$ true for this x.]
> - Thus $\lim_{x \to a} f(x) = L$.

(1) Prove that $\displaystyle\lim_{x \to 1} \frac{2x^2 - 2}{x - 1} = 4$.

(2) Let a, m, and b be real numbers. Prove[1] that $\displaystyle\lim_{x \to a} mx + b = ma + b$.

(3) Prove[2] that limits, if they exist, are unique. That is, show that if $\displaystyle\lim_{x \to a} f(x) = L$ and $\displaystyle\lim_{x \to a} f(x) = M$ are both true, then $L = M$.

[1]Hint: You might want to separate the cases withe $m = 0$ and $m \neq 0$.

[2]Hint: To obtain a contradiction, suppose that $L \neq M$. Take $\epsilon = \frac{|L-M|}{3}$ in the definition. Compare with the argument that a sequence converges to at most one value.