Side 1: **True or False. Justify.**

(1) The function $f(x) = |x^3|$ is differentiable at $x = 0$.

(2) If $f : [a, b] \to \mathbb{R}$ is continuous and f attains its maximum on $[a, b]$ at $x = c$, then $f'(c) = 0$.

(3) Every nonempty set of real numbers that is bounded above has a maximum element.

(4) The supremum of the set $\{x \in \mathbb{Q} \mid x < \pi\}$ is π.

(5) If the domain of f is $\mathbb{R}$, then f is continuous at some value of x.

(6) Every decreasing sequence is convergent.

(7) The function $f(x) = x^3 - 2x^2 + 5$ is increasing or decreasing on $(0, 3)$.

(8) The sequence $\left\{ \dfrac{\sin(n^2)}{n} \right\}_{n=1}^{\infty}$ is convergent.

(9) We can prove that every polynomial $p(x)$ has a property P by induction on degree by showing that every constant function has property P and then showing that if $p(x)$ has property P then so does $p'(x)$.

(10) For every pair of integers $m, n \in \mathbb{Z}$, $m^2 \neq 8n^2$.

(11) If f is continuous on $[1, 3]$, and $y > f(1) > f(3)$, then there is no $c \in [1, 3]$ with $f(c) = y$.

(12) If f and g are continuous on $(-7, 7)$ and $g(4) = -1$, then $\lim_{x \to 4} (f \circ g)(x) = f(-1)$.

(13) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ both diverge, then so does $\{a_n + b_n\}_{n=1}^{\infty}$.

(14) If $f(x) > 5$ for all $x \neq -7$ and $\lim_{x \to -7} f(x) = L$, then $L > 5$.

Side 2: **True or False. Justify.**

(1) The polynomial $p(x) = x^5 + 5x + 1$ has exactly one (real) root.

(2) If f is differentiable and $f'(x) > 0$ for all $x \in (a, b)$, then f is strictly increasing on (a, b).

(3) The maximum of the set $\{x \in \mathbb{Q} \mid x < \pi\}$ is π.

(4) If S is a set of real numbers and $\sup(S) \in S$, then $\sup(S)$ is the maximum element of S.

(5) Every convergent sequence is bounded.

(6) Every monotone sequence has a convergent subsequence.

(7) If f is differentiable at $x = 2$ and $f(2) = 5$, then the sequence $\left\{ f\left(\frac{2n+1}{n+4}\right) \right\}_{n=1}^{\infty}$ converges to 5.

(8) The function $f(x) = x^3 - 2x^2 + 5$ attains a minimum value on $(0, 3)$.

(9) The sequence $\left\{\sin(n^2)\right\}_{n=1}^{\infty}$ has a convergent subsequence.

(10) For any real numbers $a < b$, there is an integer c such that $a < n < b$.

(11) If f is differentiable on $\mathbb{R}$, $f'(x) \leq 0$ for all $x > 0$, and $f(x) \geq -5$ for all $x > 0$, then the sequence $\{f(n)\}_{n=1}^{\infty}$ converges.

(12) We can prove that every polynomial $p(x)$ has a property P by induction on degree by showing that every constant function has property P and then showing that if $p'(x)$ has property P then so does $p(x)$.

(13) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ both diverge to $+\infty$, then so does $\{a_n + b_n\}_{n=1}^{\infty}$.

(14) If $\lim_{x \to -3} f(x) > 5$, then $\exists \delta > 0$ such that $f(x) > 5$ for all $x \in (-3 - \delta, -3) \cup (-3, -3 + \delta)$.