Solutions

Math 325. Quiz #1

(1) State the definition of the *minimum* of a set S of real numbers.

A number m is the minimum of S provided $m \in S$ and for all $x \in S$, $m \leq x$.

(2) True or false, and justify with an argument or counterexample:

Let r be a rational number. If x is irrational, then $x + r$ is irrational.

TRUE:

If $x+r$ is rational, then $x = (x+r) - r$ is rational (since $x+r$ is & $-r$ is).

(3) True or false, and justify with an argument or counterexample:

Let S be any set of real numbers. For any $x \in S$ there is some $y \in \mathbb{R}$ such that $x < y$.

TRUE:

Given S and $x \in S$, we can take $y = x+1$.

~~**Bonus:** Prove or disprove: If S is a subset of $\mathbb{Z}$ and S has 1,000,000 as an upper bound, then S has a maximum.~~

Common mistakes:

#1: unexplained variables

e.g., [$x \in S$ and for all $y \in S$, $x \leq y$]

x is unexplained here! Correct version

explanation of "x ="

[A real number x is a minimum for S if $x \in S$ and for all $y \in S$, $x \leq y$.]

explanation of y.

e.g., [A real number x is ~~a~~ minimum for S if $x \in S$ and $y \in S$, $x \leq y$]

The y is a new variable being introduced to explain the idea that x is ~~the~~ bigger than any elt. of S... Any elt. of S; we need a quantifier.

#1: assertion vs. definition

e.g., [There is a minimum for a set S if ...]

This is about when a minimum exists, not what a minimum is.

#1(+#2 & #3): intuitive vs. precise

e.g. [The minimum of a set is the smallest element in the set.]

True, but perhaps not quite definition-grade. Unpackage "smallest in the set" in ways we can work with mathematically.

~~**Bonus:** Prove or disprove: If S is a subset of $\mathbb{Z}$ and S has 1,000,000 as an upper bound, then S has a maxi~~mum.

#2: correct negation / contrapositive

#2 & #3: argument vs. example

To show these are true we need a general argument; if they were false we could show this with a counterexample.

#3: "any set of real numbers" means an arbitrary subset of $\mathbb{R}$, not $\mathbb{R}$ itself

#2: e.g., Say r is rational, so $r = a/b$ for some $a, b \in \mathbb{Z}$, and x is irrational. Then

$$x + r = \frac{a}{b} + \frac{bx}{b} = \frac{a+bx}{b},$$

and $a + bx \notin \mathbb{Z}$, so $x + r$ is irrational.

Just because $x+r$ can be written as a fraction where one of the pieces is not an integer does not mean $x+r$ is irrational.

(e.g., $1 = \frac{\sqrt{2}}{\sqrt{2}}$)