Math 325-002 — Fake Problem Set
Due: Never

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Let $[a, b]$ be a closed interval and f be a continuous function on $[a, b]$. Show that the range of f (that is, $\{f(x) \mid x \in [a, b]\}$) is a closed interval.

(2) Let f and g be functions defined on $\mathbb{R}$ and a a real number. Assume that f is differentiable at a and $f(a) = f'(a) = 0$.
   (a) Use the product rule to show that if g is differentiable at a, then $(fg)'(a) = 0$.
   (b) Show that if g is continuous at a, then $(fg)'(a) = 0$.
   (c) Show that if g is *not* continuous at a, then fg may not be differentiable at a.

Problems that will only make sense after Tuesday:

(3) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable on $\mathbb{R}$, $f(0) = 0$, and $f'(x) < 1$ for all $x \in \mathbb{R}$. Show that $f(x) < x$ for all $x > 0$.

(4) A fixed point of a function is a function x is a number r such that $f(r) = r$. Show that if $f : \mathbb{R} \to \mathbb{R}$ is differentiable on $\mathbb{R}$ and $f'(x) \neq 1$ for all $x \in \mathbb{R}$, then f has at most one fixed point.