

# WHAT TO KNOW FOR QUIZZES AND EXAMS

## DEFINITIONS

- (1) Rational number
- (2) Contrapositive
- (3) Converse
- (4) Irrational number
- (5) Upper bound / lower bound
- (6) Bounded above / bounded below
- (7) Supremum / infimum
- (8) Absolute value
- (9) (sequence) converges to  $L$
- (10) (sequence) is convergent
- (11) (sequence) is divergent
- (12) increasing / decreasing sequence
- (13) strictly increasing / decreasing sequence
- (14) monotone sequence
- (15) diverges to  $\infty$  or  $-\infty$
- (16) Subsequence
- (17) Cauchy sequence
- (18) Limit of a function
- (19) Continuous at a point
- (20) Continuous on an open interval
- (21) Continuous on a closed interval
- (22) Differentiable
- (23) Derivative (at a point)
- (24) Derivative (function)
- (25) Increasing/decreasing function

## AXIOMS/THEOREMS

- (1) Well-ordering axiom
- (2) Completeness axiom
- (3) Theorem 7.1 (large natural numbers)
- (4) Archimedean principle
- (5) Density of rational numbers/ irrational numbers
- (6) Triangle inequality
- (7) Theorem 12.2 (limits and algebra)
- (8) Squeeze Theorem
- (9) Monotone convergence theorem
- (10) Principle of induction
- (11) Theorem 18.6 (convergence and subsequences)
- (12) Theorem 19.2 (sequence with subsequences converging to every real number)
- (13) Cantor's Theorem
- (14) Bolzano-Weierstrass
- (15) Main corollary of Bolzano-Weierstrass

- (16) Cauchy if and only if convergent
- (17) Theorem 25.1 (limits and sequences)
- (18) Theorem 26.4 (limits of functions and algebra)
- (19) Squeeze Theorem for functions
- (20) Theorem 27.5 (continuity and limits)
- (21) Theorem 28.1 (continuity and algebra)
- (22) Theorem 28.3 (continuity and compositions)
- (23) Intermediate Value Theorem
- (24) Boundedness Theorem
- (25) Extreme Value Theorem
- (26) Derivatives and algebra (Theorem 32.9)
- (27) Chain rule (Theorem 33.3)
- (28) Min-Max Theorem
- (29) Mean Value Theorem
- (30) Increasing/decreasing functions and derivatives

#### KEY SKILLS

- (1) Proving “if-then” statements, “for every” statements, “there exists” statements, “is unique” statements
- (2) Proofs by contradiction
- (3) Finding the negation of a statement
- (4) Finding the contrapositive of a statement
- (5) Using examples to prove/disprove statements
- (6) Proving that a number is the supremum of a set
- (7) Proving that a sequence converges to some value using the definition
- (8) Algebra with limits of sequences: using these to determine if a sequence converges, and to what
- (9) Using squeeze theorem to show sequences converge
- (10) Relationship between boundedness, convergence, and monotonicity
- (11) Proofs by induction
- (12) Relationship between convergence/boundedness of sequences and convergence of sub-sequences
- (13) Using the Cauchy property to show a sequence converges
- (14) Using the  $\varepsilon - \delta$  definition to compute limits
- (15) Using algebra/squeeze theorem to compute limits
- (16) Using the  $\varepsilon - \delta$  definition to show continuity
- (17) Using algebra/compositions to show continuity
- (18) Applying the  $\varepsilon - \delta$  definitions of limits and continuity
- (19) Applying the Intermediate Value Theorem
- (20) Applying Boundedness and Extreme Value Theorems
- (21) Evaluating derivatives by definition
- (22) Evaluating derivatives by algebra and chain rule
- (23) Using definition of derivative and Min-Max Theorem to determine when values of  $f$  are larger / smaller than others
- (24) Using definition of derivative and mean value theorem to determine increasing / decreasing behavior of functions