

Math 325-002 — Problem Set #6
Due: Wednesday, October 13 by 5pm

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like “I collaborated with Steven Smale on problems 1 and 3”. If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

- (1) Prove by induction on n that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

for all $n \in \mathbb{N}$.

- (2) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence and let L be a real number. Show that $\{a_n\}_{n=1}^{\infty}$ converges to L if and only if the subsequences $\{a_{2k}\}_{k=1}^{\infty}$ and $\{a_{2k-1}\}_{k=1}^{\infty}$ both converge to L .