Math 325-002 — Problem Set #1
Due: Wednesday, September 1 by 5pm

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Prove that there is no rational number whose square is 3 by mimicking[1] the proof of Theorem 1.1 from class.

(2) Prove[2] that there is no real number whose square is -1.

(3) For each of the following sets, which of the axioms of a field, listed in Theorem 1.1. of our text (page 5), do *not* hold if one replaces $\mathbb{Q}$ with the indicated set? Explain.
  (a) The set of nonnegative integers $\{0, 1, 2, 3, \dots\}$.
  (b) The set of nonnegative rational numbers $\{q \in \mathbb{Q} \mid q \geq 0\}$.
  (c) The set of all integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

(4) Prove the following "Cancellation of multiplication" property: If x, y, and z are real numbers such that $xy = xz$ and $x \neq 0$, then $y = z$. Your proof should use nothing other than the axioms of the real numbers, just as I did in lecture to show Cancellation of Addition. (You will not need to use the completeness axiom).

(5) A real number that is not rational is called *irrational*.
  (a) Prove that if x is a real number and x^2 is irrational, then x is also irrational.
  (b) Prove that the converse of the statement in (a) is false.

(6) Read Section 1.4 of our text. Then do #7 and #10 on pages 41–42. *Warning*: Be sure you are using the Second Edition of the text for these problems. If you are using the First Edition, contact me and I can give you the correct problem numbers.

(7) Do #9 and #11 on pages 41–42 of our text. *Warning*: Be sure you are using the Second Edition of the text for these problems. If you are using the First Edition, contact me and I can give you the correct problem numbers.

[1] Some steps will be the same, but there should be some slight differences!

[2] A proof along the lines of Theorem 1.1 won't work for this. Instead, you can use without proof that if $x \leq 0$ and $y \leq 0$, then $xy \geq 0$; we will discuss this fact in class.