

**Math 325-002 — Problem Set #9**  
**Due: Wednesday, November 10 by 5 pm**

**Instructions:** You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Prove that the function  $f(x) = \sqrt{|x|}$  is continuous at every real number  $a$  using just the  $\epsilon - \delta$  definition of continuity.

(2) Let  $f(x)$  be the function with domain all of  $\mathbb{R}$  defined by

$$f(x) = \begin{cases} 2 + x, & \text{if } x \leq 2 \text{ and} \\ 2x - 5, & \text{if } x > 2. \end{cases}$$

Prove  $f$  is not continuous at  $x = 2$  using just the  $\epsilon - \delta$  definition of continuity.

(3) Prove  $\sqrt{\sqrt{x^2 + 1} + x^4 + 1}$  is continuous on all of  $\mathbb{R}$ . You may use any theorems we've covered in class, but be sure to use them carefully.

(4) Suppose  $f$  is a function that is continuous on all of  $\mathbb{R}$ . Prove<sup>1</sup> that if  $f(q) = 0$  for all  $q \in \mathbb{Q}$ , then  $f(x) = 0$  for all  $x$ .

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<sup>1</sup>Hint: You may want to use that every real number is the limit of some sequence of rational numbers.