Math 325-002 — Problem Set #4
Due: Wednesday, September 22 by 5 pm

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Given any two real numbers x and y, $\max\{x, y\}$ refers to the larger of the two numbers x and y; that is, $\max\{x, y\}$ is x if $x \geq y$ and otherwise it is y. Similarly, $\min\{x, y\}$ refers to the smaller of the two numbers x and y; that is, $\min\{x, y\}$ is x if $x \leq y$ and otherwise it is y.

(a) Prove that for all real numbers x and y

$$\max\{x, y\} = \frac{x + y + |x - y|}{2}.$$

(b) Find a similar formula for $\min\{x, y\}$ and prove that your formula is correct.

(2) Prove, using the formal definition of convergence, that the sequence $\{\frac{1}{\sqrt{n}}\}_{n=1}^{\infty}$ converges to 0. (At the risk of being overly pedantic, for $n \in \mathbb{N}$, by $\sqrt{n}$ we mean the unique positive real number whose square is n. Such a number exists by the Completeness Axiom, which proved in detail in the case $n = 2$.)

(3) Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences and L be a real number. Suppose that there is some $M \in \mathbb{R}$ such that for all $n > M$, we have $a_n = b_n$. Prove that $\{a_n\}_{n=1}^{\infty}$ converges to L if and only if $\{b_n\}_{n=1}^{\infty}$ converges to L.

(4) Do problem #5 on page 91 of the textbook.

(5) Do problem #6 on page 91 of the textbook.

(6) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence, and K, L be real numbers. Suppose that for all $n \in \mathbb{N}$, $a_n \geq K$, and that $\{a_n\}_{n=1}^{\infty}$ converges to L. Prove that $L \geq K$.