Math 325-002 — Problem Set #3
Due: Wednesday, September 15 by 5 pm

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

If you do work with others, I ask that you write something along the top like "I collaborated with Steven Smale on problems 1 and 3". If you use a reference, indicate so clearly in your solutions. In short, be intellectually honest at all times.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

(1) Prove[1] that if ϵ is any real number such that $\epsilon > 0$, then there exists a natural number n such that $0 < \frac{1}{n} < \epsilon$.

(2) Let S be the set $\{1 - \frac{1}{n} \mid n \in \mathbb{N}\}$. Prove[2] that 1 is the supremum (aka least upper bound) of S.

(3) Let r be any real number. Consider the set

$$S_r = \{q \in \mathbb{Q} \mid q < r\}.$$

In words, S_r is the set of those *rational* numbers that are strictly less than r. Prove that the supremum of S_r is r.

(4) Read Section 1.7 of the text, and do problems 4 and 7 on pages 65–66.

[1]Tip: You will need to use the following fact, proven in lecture: If x is any real number, then there is a natural number n such that $n > x$.

[2]Tip: First show 1 is an upper bound, and then use a proof by contradiction. That is, assume b is an upper bound of S such that $b < 1$ and proceed to derive a contradiction. The statement proven in the previous exercise might be useful.