

Math 325-002 — Problem Set #11
Due: Wednesday, December 8 by 5 pm

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

- (1) Let $p(x)$ be a polynomial function.
 - (a) Use Rolle's Theorem to show that if $p(x)$ has n real roots, then $p'(x)$ has at least $n - 1$ real roots.
 - (b) Use Rolle's Theorem and induction to show that if $p(x)$ is not a constant function, then the number of real roots of $p(x)$ is at most the degree of p .

- (2) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable on \mathbb{R} , and assume that $f(0) = 0$, and that $f'(x) < 1$ for all $x \in \mathbb{R}$.
 - (a) Show that $f(x) < x$ for all $x > 0$.
 - (b) Does the same equality necessarily hold for $x < 0$? Justify your answer.

- (3) A *fixed point* of a function f is a number x such that $f(x) = x$. Prove that if f is differentiable on \mathbb{R} , and $f'(x) \neq 1$ for all $x \in \mathbb{R}$, then f has at most one fixed point.