

Math 325-002 — Problem Set #10
Due: Wednesday, December 1 by 5 pm

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand.

Please write neatly, using complete sentences and correct punctuation. Label the problems clearly.

- (1) Show that¹

$$f(x) = \begin{cases} x^2 - 2x & \text{if } x \geq 2 \\ 2x - 4 & \text{if } x < 2 \end{cases}$$

is differentiable at all values of x , and compute the derivative.

- (2) Show that the function

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$

is differentiable at $x = 0$, and not differentiable at any other value of x .

- (3) Let f and g be functions and a be a real number. Assume that $f(a) = f'(a) = 0$ and that g is continuous at a .

(a) Prove² that $(fg)'(a) = 0$.

(b) Is the statement still true if we do not assume that g is continuous at a ? Prove or disprove it.

¹Hint: Break into three cases.

²Note: We are not assuming that g is differentiable, so we cannot apply the product rule.