TRUE or FALSE. Justify.

(1) Let $x, y \in \mathbb{R}$. The negation of the statement "If x and y are rational, then xy is rational" is "If x and y are rational, then xy is irrational". F

(2) Let $x, y \in \mathbb{R}$. The contrapositive of the statement "If x and y are rational, then xy is rational" is "If xy is irrational, then x is irrational or y is irrational". T

(3) The commutative property/axiom of addition says that $x + y = y + x$. T

(4) Every set of real numbers that is bounded above has a supremum. F

(5) There is a set S of real numbers such that $\sup(S)$ exists, but $\sup(S) \notin S$. T

(6) If $a < b$ are real numbers, there is an integer $n \in \mathbb{Z}$ such that $a < n < b$. F

(7) Every nonempty set of real numbers has a smallest element (i.e., a minimum element). F

(8) Every nonempty set of integers that is bounded below has a smallest element (i.e., a minimum element). T

(9) If $S \subseteq \mathbb{R}$ is bounded above, then there is a natural number b such that b is an upper bound for S. T

(10) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $x < y^2$". T

(11) Every set of real numbers satisfies the property that "for all $x \in S$, there exists a real number y such that $y^2 < x$". F

(12) The supremum of the set $\{1/n \mid n \in \mathbb{N}\}$ is 1. T

(13) The supremum of the set $\{-1/n \mid n \in \mathbb{N}\}$ is -1. F

(14) The negation of the statement "for all $x \in S$, there exists a real number y such that $x < y^2$" is "for all $x \in S$, there exists a real number y such that $x \geq y^2$". F

(15) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to 5, then for all natural numbers n, $a_n > 4$. F

(16) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to L, then there is some $N \in \mathbb{R}$ such that for all natural numbers $n > N$, $a_n = L$. F

(17) For every real number L there is a sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_n \neq L$ for all $n \in \mathbb{N}$ and converges to L. T

(18) A sequence of positive numbers can converge to a negative number. F

(19) A sequence of positive numbers can converge to zero. T

(20) There is a set S of irrational numbers such that $\sup(S) = 2$. T

(21) Every increasing sequence is convergent. F

(22) Every convergent sequence is either increasing or decreasing. F

(23) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a convergent sequence. T

(24) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent sequences, and $b_n \neq 0$ for all $n \in \mathbb{N}$, then $\{a_n/b_n\}_{n=1}^{\infty}$ is a convergent sequence. F

(25) The sequence $\left\{\dfrac{3n^2 - 4n + 7}{6n^2 + 1}\right\}_{n=1}^{\infty}$ converges to $1/2$. T

(26) The negation of "$\{a_n\}_{n=1}^{\infty}$ is a monotone sequence" is "there exists $n \in \mathbb{N}$ such that $a_n > a_{n+1}$ and $a_n < a_{n+1}$". F

(27) Every convergent sequence of rational numbers converges to a rational number. F

(28) If a sequence is not bounded below, then it diverges to $-\infty$. F

(29) If $\{a_n\}_{n=1}^{\infty}$ diverges to ∞ and $\{b_n\}_{n=1}^{\infty}$ diverges to $-\infty$, then $\{a_n + b_n\}_{n=1}^{\infty}$ converges to 0. F

(30) If $\{a_n\}_{n=1}^{\infty}$ diverges to ∞ and $\{b_n\}_{n=1}^{\infty}$ converges, then $\{a_n + b_n\}_{n=1}^{\infty}$ diverges to ∞. T

(31) If $\{a_n^2\}_{n=1}^{\infty}$ converges to 1, then $\{a_n\}_{n=1}^{\infty}$ converges. F