A. 1) OK

2) zero polynomial 0 $(= 0t^n + 0t^{n-1} + \cdots + 0)$

3) ~~no~~, since 0 is not in the set

4) yes : 0 is in, $p(1)=0,\ q(1)=0 \Rightarrow (p+q)(1)=0$

$p(1)=0 \Rightarrow (Cp)(1)=0.$

B. 1) Can't ~~add~~ matrices of different sizes.

2) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

3) set of diagonal matrices

4) it's the span of a set of vectors

C. 1) no, not closed under scalar mult.

2) if $y = 2x$, then $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 2x \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\in \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}.$

D. 1) any element of P_2 is

$a + bt + ct^2 = a \cdot 1 + b \cdot t + c \cdot t^2$

some a, b, c.

2) if $a \cdot 1 + b \cdot t + c \cdot t^2 = 0$ (0 poly)
then $a, b, c = 0$.

3) ✓

E. 1) Spans, since any $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
is $a \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \cdots + d \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Lin. indep't, since $a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \cdots + d \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow a = b = c = d = 0$.

2) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

F. 1) LI & span ok.

2) ~~put~~ into a matrix. Pivot in every row $\Rightarrow$ Spans. Pivot in every col $\Rightarrow$ LI.

3) Yes; see (2).

6. 1) $x_1 = 2x_3 - 2x_5$

$x_2 = -2x_3$

$x_4 = x_5$

2) $x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \rightsquigarrow \left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$

3) $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \right\}.$

H. 1) $z(t)$ zero poly $\rightsquigarrow z(\pm 1) = 0.$

$p(\pm 1) = 0,\ q(\pm 1) = 0 \Rightarrow (p+q)(\pm 1) = 0.$

$p(\pm 1) = 0 \Rightarrow cp(\pm 1) = 0.$

2) $\{t^2 - 1,\ t^3 - t\}.$

I. 1) $\underline{0} \in H$, $\underline{0} \in K \Rightarrow \underline{0} \in H \cap K$

$\underline{v}, \underline{w} \in H \cap K \Rightarrow \underline{v}, \underline{w} \in H \Rightarrow \underline{v}+\underline{w} \in H$

$\Rightarrow \underline{v}, \underline{w} \in K \Rightarrow \underline{v}+\underline{w} \in K \Rightarrow \underline{v+w} \in H \cap K$.

2) Generally not: $\mathrm{Span}\{\begin{bmatrix}1\\0\end{bmatrix}\} \cup \mathrm{span}\{\begin{bmatrix}0\\1\end{bmatrix}\} \subseteq \mathbb{R}^2$ is not closed under $+$.

3) Check it.

4) No. x-axis $\subseteq \mathbb{R}^2$ is a subspace. Can take $\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\}$ as a basis for $\mathbb{R}^2$.

5) Yes. S is a LI subset of $\mathbb{R}^n$. Suppose there is no vector $\underline{v} \in \mathbb{R}^n$ such that $S \cup \{\underline{v}\}$ is LI. Then $\underline{v} \in \mathrm{Span}(S)$ for all $\underline{v} \in \mathbb{R}^n$, so $H = \mathbb{R}^n$. Thus, if $H \subsetneq \mathbb{R}^n$, we can extend S to a larger LI subset. Keep doing this

until we have a LI subset of size n. Then this must be a basis for $\mathbb{R}^4$.

J. 1) a) yes

b) no

c) no

d) yes

2) a) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$.