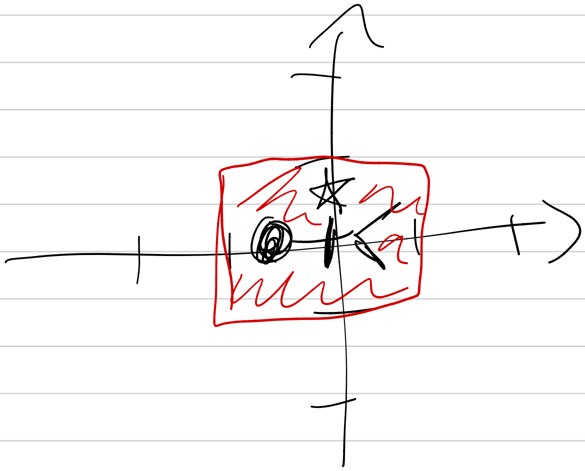
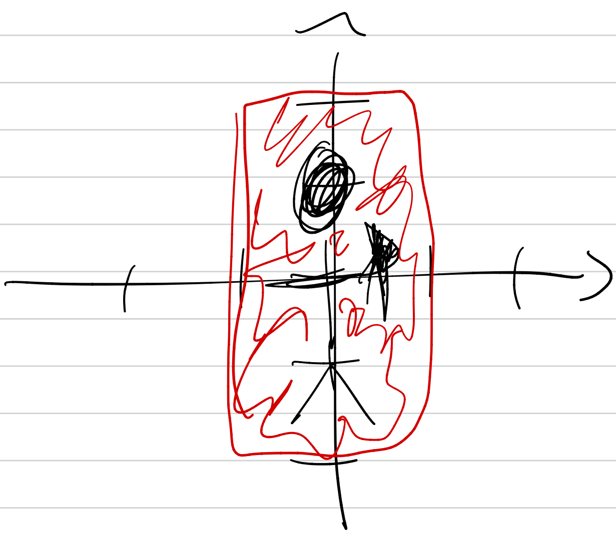
Worksheet #4 solutions

A 1)



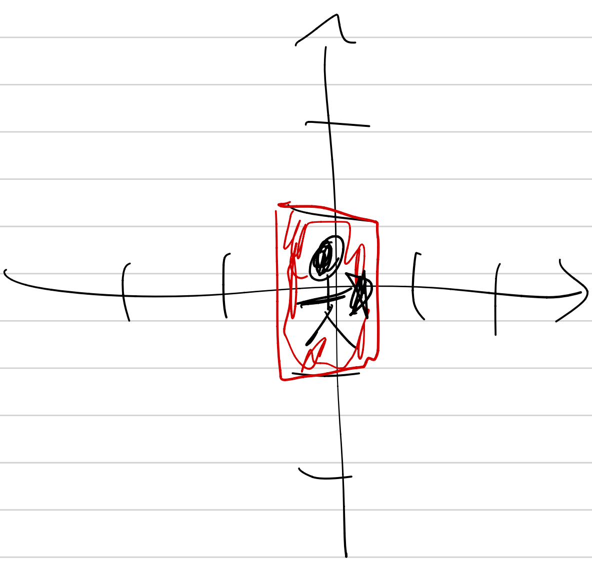
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

2)



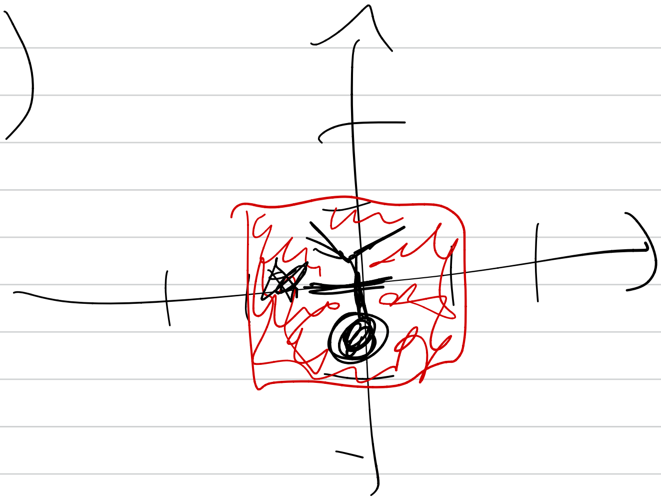
$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

3)



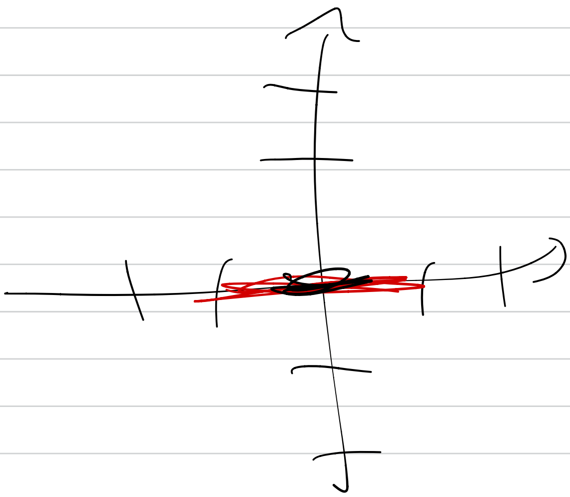
$$\begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$$

4)



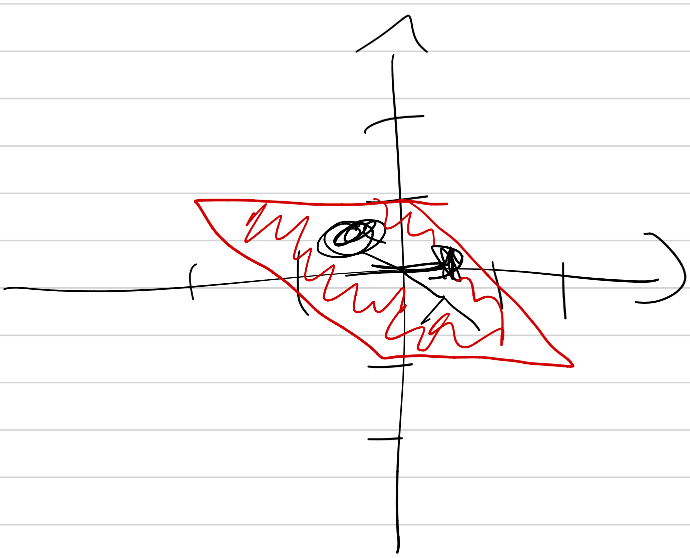
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

5)



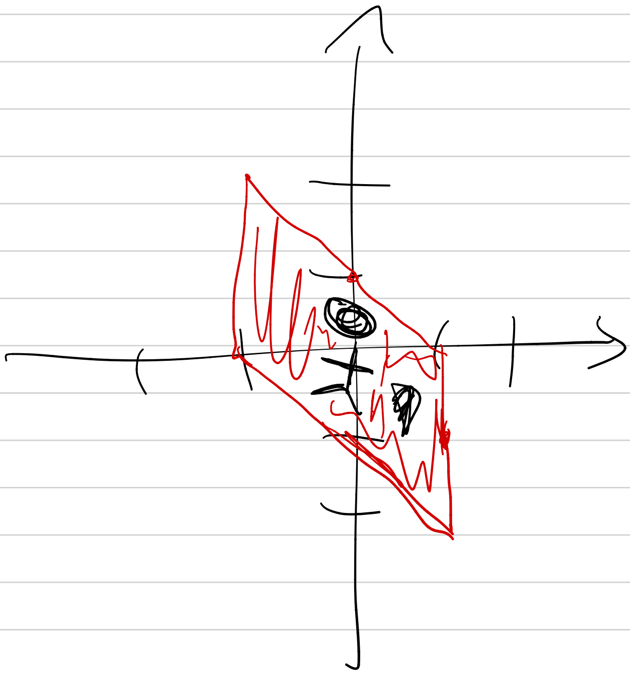
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

B. 1)



$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

2)



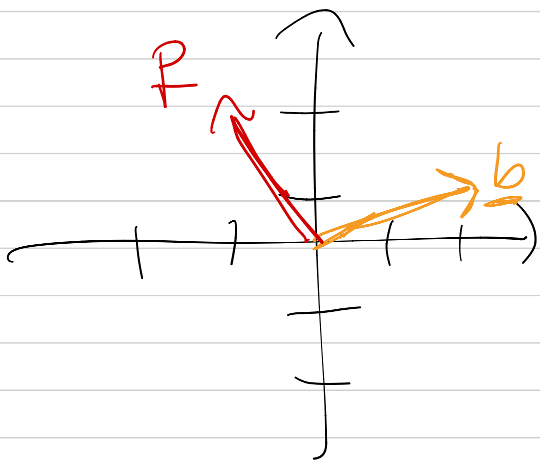
$$\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

3)

$$\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

C. 1) $\mathbb{R}^2$; $\mathbb{R}^2$

2)



3) yes ; onto

4) no ; one-to-one

D. 1) $\mathbb{R}^2$; $\mathbb{R}^2$

2) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$; no

3) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$; no

(there are many answers)

E. 1) $\mathbb{R}^5$; $\mathbb{R}^4$

2) A is row equivalent to

$$\begin{bmatrix} 1 & -1 & 3 & -4 & 5 \\ 0 & 2 & 1 & 0 & 3 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \Rightarrow \text{onto but not one-to-one}$$

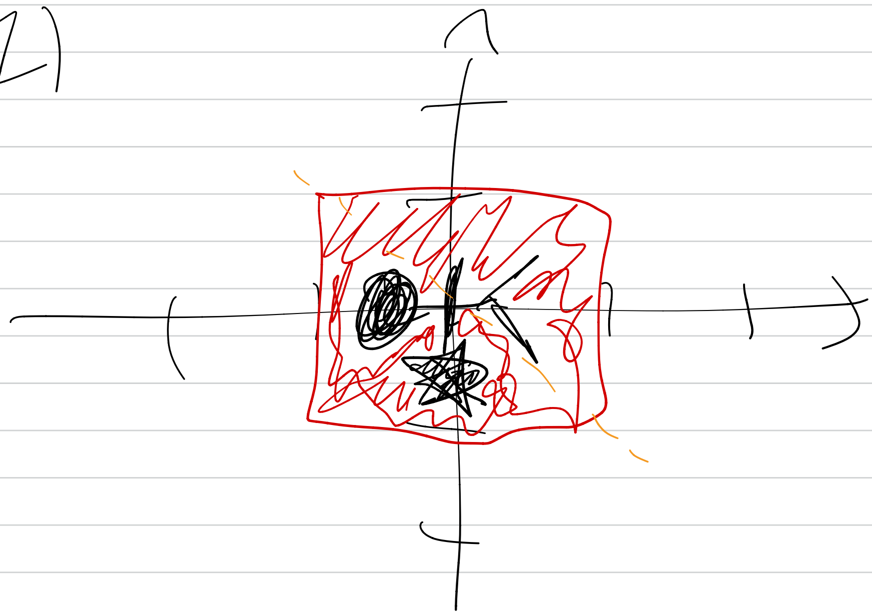
3) $A\underline{x} = \underline{b}$ is always consistent, never unique solution.
Every solution ~~set~~ is a line.

F. 1) no

2) yes: 2×2

3) yes: 5×5

6. 1)



reflection over $y=-x$

2) $$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

3) $$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

4) $$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

H. 1) From the theorem on the first page about standard matrices.

2) $(T \circ U)(\underline{e_1}) = T(U(\underline{e_1}))$

$$= T(\underline{b_1}) = A\underline{b_1}.$$

3) Similarly,

$(T \circ U)(\underline{e_i}) = A\underline{b_i}$ for each i.

So standard matrix of $T \circ U$ is

$$\begin{bmatrix} (T \circ U)(\underline{e_1}) & \cdots & (T \circ U)(\underline{e_n}) \end{bmatrix}$$

$$= \begin{bmatrix} A\underline{b_1} & \cdots & A\underline{b_n} \end{bmatrix}.$$

I 1) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

2) $AB = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

rotation around line $x=y=z$!