Week 3 worksheet solutions

A.

$$\underline{x} = \begin{bmatrix} x_4 - 3x_5 + 2 \\ -x_4 - 3 \\ x_3 \\ x_4 \\ x_5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

B. 1)

$$\begin{bmatrix} 3 \\ -3/5 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ -1/5 \\ 1 \end{bmatrix} \quad s \in \mathbb{R}$$

line in $\mathbb{R}^3$

2) many possible answers, e.g.,

$$\begin{bmatrix} -1 \\ -4/5 \\ 1 \end{bmatrix} + s \begin{bmatrix} -4 \\ -1/5 \\ 1 \end{bmatrix} \quad s \in \mathbb{R}$$

3)

$$s \begin{bmatrix} -4 \\ -1/5 \\ 1 \end{bmatrix} \quad s \in \mathbb{R}$$

4) $$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -4 \\ -1/5 \\ 1 \end{bmatrix} \quad s \in \mathbb{R}.$$

C. 1) linearly dependent

$$\underline{v_3} = -2/4\, \underline{v_1} + 3/4\, \underline{v_2}.$$

2) $$\begin{bmatrix} \underline{v_1} & \cdots & \underline{v_t} \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_t \end{bmatrix} = \underline{0}.$$ yes

3) nontrivial

D. 1) dependent

2) independent

3) dependent

4) dependent

5) independent

6) dependent

7) independent

8) dependent

E. 1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

3) $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$ $m \geq n$

4) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & a \\ 0 & 0 \end{bmatrix}$
$a \in \mathbb{R}$

5) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$a \in \mathbb{R}$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 0 \end{bmatrix}$
$a, b \in \mathbb{R}$ $a, b \in \mathbb{R}$

(I think I got them all...)

F. 1) $T\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2T\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow T$ is <u>not</u> linear.

2) $T(\underline{u} + \underline{v}) = -(\underline{u} + \underline{v}) = -\underline{u} + -\underline{v}$

$= T(\underline{u}) + T(\underline{v})$

$$T(c\underline{u}) = -(c\underline{u}) = c \cdot -\underline{u} = c\,T(\underline{u}).$$

Yes it is linear.

3) $$T\left(\begin{bmatrix}2\\2\end{bmatrix}\right) = T\left(2\begin{bmatrix}1\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\end{bmatrix}\right)$$

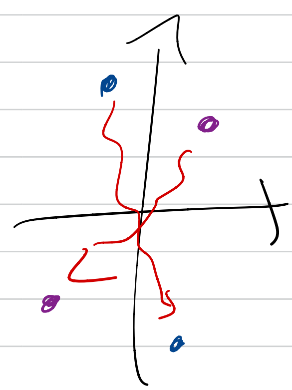
$$= 2\,T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) + 2\,T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$

$$= 2\begin{bmatrix}-1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix}.$$

6.1) $$T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}-x\\-y\end{bmatrix}$$

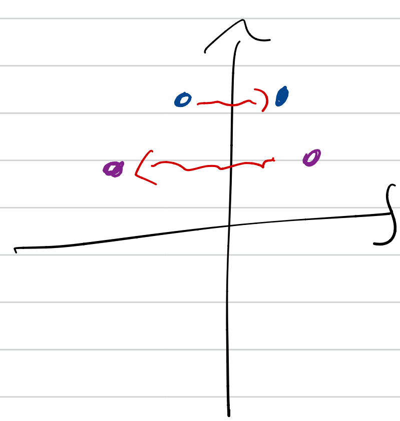
point goes to its negative

$\equiv$ 180° rotation



2) $$T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}-x\\y\end{bmatrix}$$

reflection over y-axis.



3) $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} y \\ z \end{bmatrix}$

projection onto yz-plane

4. 1) Set of the form $\underline{p} + s\underline{v}$ $s \in \mathbb{R}$. $\underline{v} \neq \underline{0}$.

$T(\underline{p} + s\underline{v}) = T(\underline{p}) + sT(\underline{v})$ for any $s \in \mathbb{R}$.

If $T(\underline{v}) \neq \underline{0}$, this is a line.

If $T(\underline{v}) = \underline{0}$, it's just the point $T(\underline{p})$.

2) No: in G3, $T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$ lin indep't, but $\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$ lin dep't.

3). Yes: if $\underbrace{c_1\underline{v_1} + \cdots + c_t\underline{v_t} = \underline{0}}_{\text{with some } c_i \neq 0}$, then

$\underline{0} = T(\underline{0}) = T(c_1\underline{v_1} + \cdots + c_t\underline{v_t})$

$= c_1 T(\underline{v_1}) + \cdots + c_t T(\underline{v_t})$

4) · Square → parallelogram,
line segment, or point

· triangle → triangle,
line segment, or point

· Circle → ellipse, line segment, or point