

Math 314. Review questions

- Basics of linear systems
 - What is a linear equation?
 - What is a linear system?
 - What is a solution of a linear system?
 - What is the solution set of a linear system?
 - What are the possible different numbers of solutions of a linear system?
 - What is the augmented matrix of a linear system?
 - What is the coefficient matrix of a linear system?
 - What is echelon form?
 - What is row reduced echelon form?
 - What is a pivot?
 - How can you tell how many pivots a linear system has?
 - How do you solve a linear system?
 - How can you tell if a linear system is consistent?
 - How can you tell if a linear system has more than one solution?
 - What is a free variable?
 - How do you rewrite a linear system as a vector equation?
 - How do you rewrite a linear system as a matrix-times-vector equation?
 - How can you tell if $Ax = b$ has a solution for every b ?
 - How can you tell if $Ax = b$ has at most one solution for every b ?
 - How can you tell if $Ax = b$ has exactly solution for every b ?
 - Does $Ax = 0$ always have a solution?
 - Can $Ax = b$ have a solution for some b and $Ax = c$ have no solution for some other c ?
 - How are the numbers of solutions of $Ax = b$ and $Ax = c$ related?
 - How are the shapes of the solution sets of $Ax = b$ and $Ax = c$ related?
 - If you have two solutions of $Ax = b$, how can you find a solution of $Ax = 0$?
 - If you have a solution of $Ax = b$ and a solution of $Ax = 0$, how can you find another solution of $Ax = 0$?
 - What does homogeneous mean?
 - What does equivalent matrices mean?
 - How is the RREF of an augmented matrix a system related to the RREF of the coefficient matrix of the system?
 - What are elementary row operations?
- Vectors in \mathbb{R}^n
 - What is \mathbb{R}^n ?
 - What is a vector in \mathbb{R}^n ?
 - What are the two basic operations we can do on vectors in \mathbb{R}^n ?
 - What does vector addition look like geometrically?
 - What does scalar multiplication look like geometrically?
 - What does the linear combination of v_1, \dots, v_n with weights c_1, \dots, c_n mean?
 - What does it mean to be a linear combination of v_1, \dots, v_n ?
 - What does the span of a set of vectors mean?
 - What does the span of one nonzero vector look like?
 - What are the possibilities for what the span of two nonzero vectors looks like?
 - What does solving a linear system have to do with linear combinations?
 - What does a consistency of a linear system have to do with span?

- How do you decide if a vector is in the span of a set of some other vectors?
- How do you decide if a set of vectors spans \mathbb{R}^n ?
- What is the zero vector $\mathbf{0}$ in \mathbb{R}^n ?
- What are the standard vectors \mathbf{e}_i in \mathbb{R}^n ?
- What does it mean for a set of vectors in \mathbb{R}^n to be linearly independent?
- How can you tell geometrically if a set with one vector in \mathbb{R}^n is linearly independent?
- How can you tell geometrically if a set with two vectors in \mathbb{R}^n is linearly independent?
- How can you tell if a general set of vectors in \mathbb{R}^n is linearly independent?
- What does linear independence of a set of vectors in \mathbb{R}^n have to do with span?
- What sizes of vectors and matrices can be multiplied together, and what size is the result?
- How do you compute a matrix times a vector?
- What does a matrix times a vector mean?
- What is parametric vector form?
- How do you write the solution set of a linear system in parametric vector form?
- Linear transformations in \mathbb{R}^n
 - What is a linear transformation?
 - How do you check if a function from $\mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear transformation?
 - How do you use the fact that a function is a linear transformation to find some values of the function from other values?
 - What is a matrix transformation?
 - How do you find the standard matrix of a linear transformation from geometric information about the transformation?
 - How do figure out what a linear transformation is geometrically from its standard matrix?
 - What is a rotation in \mathbb{R}^2 ?
 - What is horizontal stretching in \mathbb{R}^2 ?
 - What is vertical stretching in \mathbb{R}^2 ?
 - What is reflection over a line in \mathbb{R}^2 ?
 - What is projection onto the x or y -axis in \mathbb{R}^2 ?
 - What is a horizontal/vertical shear?
 - What is the domain of a linear transformation?
 - What is the codomain of a linear transformation?
 - What is the range of a linear transformation?
 - What does it mean for a linear transformation to be one-to-one?
 - What does it mean for a linear transformation to be onto?
 - How can you find the domain of a linear transformation from its standard matrix?
 - How can you find the codomain of a linear transformation from its standard matrix?
 - How can you express the range of a linear transformation in terms of the columns of its standard matrix?
 - How can you determine if a linear transformation is one-to-one from its standard matrix?
 - How can you determine if a linear transformation is onto from its standard matrix?
- Matrix operations and inverses
 - What operations can we do with matrices?
 - How do you add matrices?
 - Which sizes of matrices are compatible for addition?

- How do you multiply matrices by scalars?
- How do you multiply two matrices?
- What does matrix multiplication do in terms of the columns of the matrix on the right?
- What sizes of matrices are compatible for matrix multiplication?
- Is $A(B + C) = AB + AC$ for matrices?
- Is $AB = BA$ for matrices?
- Is $A(BC) = (AB)C$ for matrices?
- Does $AC = BC$ imply $A = B$ for matrices?
- Can you divide by a matrix?
- What does matrix multiplication have to do with composition of functions?
- What is the identity matrix?
- What is the transpose of a matrix?
- How is transposing matrices related to matrix multiplication?
- What does it mean for a matrix to be the inverse of another matrix?
- How do you check if a matrix is the inverse of another matrix?
- How do you find the inverse of a 2×2 matrix?
- How do you find the inverse of a larger matrix?
- What is the inverse of a linear transformation?
- How can you use the inverse of a matrix to solve a linear system?
- How can you recognize when a pair of linear transformations are inverse to one another?
- How is inverting matrices related to matrix multiplication?
- What does it mean for a matrix to be invertible?
- How can you check if a matrix is invertible?
- What can you say about the solution sets of $Ax = b$ if A is invertible?
- Can you determine whether A is invertible from the solution sets of $Ax = b$ for various b ?
- What does invertibility have to do with pivots?
- What does invertibility have to do with linear independence of columns of a matrix?
- What does invertibility have to do with the span of the columns of a matrix?
- Determinants
 - What is the determinant of a 1×1 matrix?
 - What is the determinant of a 2×2 matrix?
 - What is a cofactor expansion along the first row?
 - What is a cofactor expansion along a row?
 - What is a cofactor expansion along a column?
 - What is the determinant of a diagonal matrix?
 - What is the determinant of a triangular matrix?
 - How does the determinant change with row replacement?
 - How does the determinant change with row swapping?
 - How does the determinant change with row rescaling?
 - How is the determinant of a matrix related to invertibility?
 - How is the determinant of a matrix related to the determinant of its transpose?
 - How is the determinant of a product of matrices related to the two determinants?
 - Is the determinant of a sum of matrices related to the sum of the determinants?
- Vector spaces
 - What two operations does a vector space have?
 - What does it mean to be a vector in a general vector space?

- What does it mean to be the zero vector in a vector space?
- Come up with five different types of examples of vector spaces.
- What is a subspace of a vector space?
- How do you check if a subset of a vector space is a subspace?
- What is a linear transformation between two vector spaces?
- How do you check if a function between two vector spaces is a linear transformation?
- Come up with five different types of examples of linear transformations between vector spaces other than \mathbb{R}^n .
- What is the kernel of a linear transformation?
- What do kernel and range have to do with subspaces?
- What two important subspaces are associated to any matrix?
- What does column space have to do with range?
- What does null space have to do with kernel?
- What is the span of a set of vectors in a vector space?
- What does span have to do with subspaces?
- What does it mean for a set of vectors to span a vector space?
- What does it mean for a set of vectors to be linearly independent in a vector space?
- What is a basis of a vector space?
- How do you find a basis for the columns space of a matrix?
- How do you find a basis for the null space of a matrix?
- What is the dimension of a vector space?
- How do you find the dimension of a vector space, given a basis?
- How big can a linearly independent set in a vector space be, if we know the dimension?
- How small can a set that spans a vector space be, if we know the dimension?
- How do you find the dimension of the null space of a matrix?
- How do you find the dimension of the column space of a matrix?
- Coordinates
 - What does it mean to be the vector of \mathcal{B} -coordinates of a vector in a vector space, given a basis \mathcal{B} ?
 - How do you find the vector of \mathcal{B} -coordinates of a vector in a vector space?
 - How do you find a vector in a vector space, given the vector of \mathcal{B} -coordinates?
 - How do you find a vector in a subspace of \mathbb{R}^n , given the vector of \mathcal{B} -coordinates for some basis \mathcal{B} ?
 - How do you find the vector of \mathcal{B} -coordinates of a vector in a subspace of \mathbb{R}^n , given some basis \mathcal{B} ?
 - What is the change of coordinates matrix for a basis \mathcal{B} of \mathbb{R}^n ?
 - How do you compute the change of coordinates matrix for a basis \mathcal{B} of \mathbb{R}^n ?
 - What is the change of coordinates matrix for two bases \mathcal{B}, \mathcal{C} of a vector space?
 - How do you compute the change of coordinates matrix for two bases \mathcal{B}, \mathcal{C} of a vector space?
 - What is the \mathcal{B} -matrix of a linear transformation from a vector space to itself, for a basis \mathcal{B} ?
 - How do you compute the \mathcal{B} -matrix of a linear transformation from a vector space to itself, for a basis \mathcal{B} ?
 - If \mathcal{B}, \mathcal{C} are two bases of a vector space, how are the \mathcal{B} -matrix and \mathcal{C} -matrix of the same linear transformation related?

- If \mathcal{B} is a basis of \mathbb{R}^n , and T is a linear transformation from $\mathbb{R}^n \rightarrow \mathbb{R}^n$, how is the standard matrix of T related to the \mathcal{B} -matrix of T ?
- Eigenvectors
 - What is an eigenvector of a matrix?
 - What is an eigenvalue of a matrix?
 - What is the characteristic polynomial of a matrix?
 - How do you compute the eigenvalues of a matrix?
 - How do you compute the eigenspaces of a matrix?
 - How do you recognize eigenvectors of a matrix from its associated linear transformation?
 - Does every square matrix have an eigenvector?
 - What is a diagonalizable matrix?
 - What does diagonalizability have to do with eigenvectors?
 - How do you determine if a matrix is diagonalizable?
 - What does diagonalizability have to do with having a full set of distinct eigenvalues?
 - How do you interpret diagonalizability in terms of \mathcal{B} -matrices?
 - How can you use a diagonalization to understand powers of a matrix?
- Orthogonality
 - What does it mean for two vectors to be orthogonal, geometrically?
 - How do you check if a pair of vectors is orthogonal?
 - What is an orthogonal set?
 - What is an orthonormal set?
 - What is an orthogonal basis for a subspace of \mathbb{R}^n ?
 - What is an orthonormal basis for a subspace of \mathbb{R}^n ?
 - How do you find an orthogonal basis for a subspace of \mathbb{R}^n ?
 - What is the orthogonal complement of a subspace of \mathbb{R}^n ?
 - What is the projection of a vector onto a subspace?
 - How do you compute the projection of a vector onto a subspace?
 - What does projection have to do with closest points?
 - How do you find the standard matrix of projection onto a subspace?
 - What is the orthogonal complement of the null space of a matrix?
 - What is a least-squares solution to a linear system?
 - How are least-squares solutions related to projections?
 - When does a linear system have at least one least-squares solution?
 - When does a linear system have at most one least-squares solution?