Linear Algebra (Math 314) – Project 2

Due Friday, November 4

Instructions: The following project is based on sections 3.3, 5.6 of our text, and our own everyday lives. It covers a few elementary applications of linear systems, most of which were not covered in the lectures. You should carefully read through the sections and pay particular attention to the examples presented in the text. After reading the sections, complete the three problems included below. *You must show all relevant work on all problems in order to get full credit.* You may use a computer or calculator to compute the RREF of matrices. You may work in groups of 2–3 students and you need only submit one project per group, with all the names of the group members at the top.

1. **Determinants and Geometry** – Section 3.3

   (a) Let S be the parallelogram with vertices $(2,0)$, $(7,1)$, $(0,2)$, and $(5,3)$ in $\mathbb{R}^2$. Find the area of S using determinants.
   Hint: you will first want to find the vectors that represent the edges of the parallelogram.

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by the standard matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$. Compute the area of the image of S under T.

   (c) Suppose a rectangle R has vertices $(0,0)$, $(0,5)$, $(x,0)$, and $(x,5)$. If the area of the image of R under T is found to be 10, what is the value of x?

   (d) Use determinants to find the volume of the parallelepiped in $\mathbb{R}^3$ with one vertex at the origin and adjacent vertices at $(1,2,-1)$, $(2,3,2)$, and $(0,5,5)$.

2. **Discrete Dynamical Systems** – Section 5.6

   A Canadian ecosystem consists of 3 species of animals: two main predators, the lynx and the fox; and their favorite prey that they compete for, the snowshoe hare. The evolution of this ecosystem can be modeled by the equation $\mathbf{x_{k+1}} = A\mathbf{x_k}$, where $\mathbf{x_k}$ is a vector in $\mathbb{R}^3$ representing the size of the populations for each of the three species (fox, lynx, hare) k years into the future and A is a 3×3 matrix . Let $\mathbf{x_0} = \begin{bmatrix} 190 \\ 40 \\ 790 \end{bmatrix}$ be the vector representing the three species' initial populations: 190 foxes, 40 lynx, 790 hare. Suppose that A has eigenvalues $\lambda_1 = 3$, $\lambda_2 = .8$, and $\lambda_3 = .6$ with corresponding eigenvectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} -3 \\ -3 \\ 7 \end{bmatrix}$, respectively.

   (a) Compute the vector $\mathbf{x_1}$ representing the three populations after 1 year has passed.
   Hint: This can be done either by finding the matrix A from the given information and substituting into $\mathbf{x_1} = A\mathbf{x_0}$ or by first expressing $\mathbf{x_0}$ as a linear combination of vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ and using the fact that these are eigenvectors for A. For the latter method you do not need to know what A is explicitly.

   (b) Find an explicit formula for $\mathbf{x_k}$ which involves k and the eigenvectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$.

   (c) Describe what happens to $\mathbf{x_k}$ as $k \to \infty$. Explain what your answer means in terms of the future of the three species.

(d) Is the origin an attractor, repeller, or saddle point for $\mathbf{x}_{k+1} = A\mathbf{x}_k$? Explain your answer.

3. **Find your own application.**

   Find and describe an application related to your own everyday life that can be explained with linear algebra. Your application should

   - involve a matrix with at least 3 rows and at least 3 columns
   - use numbers that are realistic for the application, and
   - utilize a technique discussed in this class other than just solving a linear system.