

# Math 314. Concepts and skills

## Chapter 1.

### *Concepts.*

- linear system
- homogeneous linear system
- solution / solution set
- $\mathbb{R}^n$
- linear combination of a set of vectors in  $\mathbb{R}^n$
- span of a set of vectors in  $\mathbb{R}^n$
- matrix-times-vector multiplication / the purpose of a matrix
- pivots of a matrix
- linear independence in  $\mathbb{R}^n$
- linear transformations  $\mathbb{R}^n \rightarrow \mathbb{R}^m$
- relationship between a linear transformations  $\mathbb{R}^n \rightarrow \mathbb{R}^m$  and matrices
- one-to-one function
- onto function
- domain and codomain of a function
- range of a function
- relationship between range of a linear transformation and standard matrix

### *Skills.*

- row reduce a matrix
- solve a linear system
- express solutions of linear systems in parametric vector form
- determine if a linear system  $A\mathbf{x} = \mathbf{b}$  has at most one solution for every  $\mathbf{b}$
- determine if a linear system  $A\mathbf{x} = \mathbf{b}$  has at least one solution for every  $\mathbf{b}$
- relate information about the solution set of the homogeneous equation  $A\mathbf{x} = \mathbf{0}$  to the nonhomogeneous equation  $A\mathbf{x} = \mathbf{b}$
- express the solution set of a homogeneous linear system as the span of a set of vectors
- determine if a set of vectors in  $\mathbb{R}^n$  is linearly independent
- determine if a set of vectors in  $\mathbb{R}^n$  spans  $\mathbb{R}^n$
- find the standard matrix of a linear transformation  $\mathbb{R}^n \rightarrow \mathbb{R}^m$
- recognize familiar linear transformations  $\mathbb{R}^n \rightarrow \mathbb{R}^m$  from their standard matrices
- determine if a linear transformation  $\mathbb{R}^n \rightarrow \mathbb{R}^m$  is one-to-one
- determine if a linear transformation  $\mathbb{R}^n \rightarrow \mathbb{R}^m$  is onto

## Chapter 2.

### *Concepts.*

- matrix multiplication
- properties of matrix multiplication
- composition of linear transformations and relationship to matrix multiplication
- inverse of a matrix
- inverse of a linear transformation
- invertible matrix
- relationship between invertibility and solution sets of  $A\mathbf{x} = \mathbf{b}$
- relationship between invertibility and number of pivots
- invertible linear transformation

*Skills.*

- multiply matrices and determine when two matrices can be multiplied
- determine if a matrix is invertible, given its entries
- determine if a matrix is invertible, given information about the solutions of  $A\mathbf{x} = \mathbf{b}$
- determine if a matrix is invertible, given information about span / linear independence of the columns
- transpose a matrix
- compute the inverse of an invertible matrix

**Chapter 3.**

*Concepts.*

- determinant of a matrix
- determinant of a matrix and row operations
- determinant of a matrix and invertibility
- determinants of products of matrices

*Skills.*

- compute the determinant using cofactor expansion
- compute the determinant using row reduction
- use determinants to determine invertibility

**Chapter 4.**

*Concepts.*

- vector space
- subspace of a vector space
- null space of a matrix
- column space a matrix
- linear combination in a general vector space
- span in a general vector space
- linear independence in a general vector space
- basis of a vector space
- dimension of a vector space
- relationship between linear independence, span, and dimension
- linear transformations between two vector spaces
- kernel of a linear transformation
- range of a linear transformation
- coordinates with respect to a basis
- change-of-coordinates matrix

*Skills.*

- determine if a subset of a vector space is a subspace
- determine if a map between vector spaces is a linear transformation
- find the dimension of a vector space, given a basis
- find the dimension of the null space of a matrix
- find a basis for the null space of a matrix
- find the dimension of the column space of a matrix
- find a basis for the column space of a matrix
- turn information about rank into information about nullity, and vice versa
- find the  $\mathcal{B}$ -coordinates of an element in a vector space
- find an element in a vector space, given its  $\mathcal{B}$ -coordinates
- find the change-of-coordinates matrix between two bases
- use the change-of-coordinates matrix to change coordinates between two bases

## Chapter 5.

### *Concepts.*

- eigenvectors
- eigenvalues
- characteristic polynomial of a matrix
- diagonalization
- $\mathcal{B}$ -matrix

### *Skills.*

- find the eigenvectors and eigenvalues of a matrix
- determine if a matrix is diagonalizable
- diagonalize a diagonalizable matrix
- use diagonalization to describe behavior of  $A^n$  for large  $n$
- compute the  $\mathcal{B}$ -matrix of a linear transformation
- use the  $\mathcal{B}$ -matrix of a linear transformation to compute or describe a linear transformation

## Chapter 6.

### *Concepts.*

- dot products
- orthogonal vectors
- orthogonal complement
- orthogonal sets
- orthonormal sets
- orthonormal basis
- orthogonal projection
- least-squares solutions

### *Skills.*

- compute the dot product of two vectors
- determine if a pair / set of vectors is orthogonal / orthonormal
- use Gram-Schmidt to turn a set into an orthogonal / orthonormal set
- use an orthogonal / orthonormal basis to compute projections
- find the least-squares solution to a linear system

## Chapter 7.

### *Concepts.*

- symmetric matrix
- spectral theorem
- singular values
- singular value decomposition

### *Skills.*

- find an orthogonal diagonalization of a symmetric matrix
- compute a singular value decomposition