Sheet 14 Solutions

A. 1) F 2) F 3) F 4) T

5) T 6) T 7) F 8) T

9) T 10) F

B. 1) $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$, $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$

2) $\begin{bmatrix}2\\5/2\end{bmatrix}$, $\begin{bmatrix}-4\\-3/2\end{bmatrix}$, $\begin{bmatrix}2 & 3\\5/2 & 2\end{bmatrix}$

3) no: $T(\underline{0}) \neq \underline{0}$

4) no; $\begin{bmatrix}-1 & 1/2\\7/2 & 1\end{bmatrix}$

C. 1) addition & scalar multiplication

2) $\mathbb{R}^n$, P_n polynomials, $M_{m\times n}$ matrices,
$Fun(\mathbb{R},\mathbb{R})$ functions

3) zero vector

D. 1) contains zero vector,
is closed under addition,
is closed under scalar multiplication

2) Yes; there is Span{$\underline{v}, \underline{w}$}

3) No: $\underline{v} \in \{s\underline{v} \mid s \geq 0\}$

$-1 \cdot \underline{v} \notin \{s\underline{v} \mid s \geq 0\}$

so it is not closed under scalar multiplication

4) ∞ 5) 1

E. 1) no; ? ; n?

2) at most 3; find a basis

4) ∞ many

F. 1) $3x_1 - 2x_2 + x_3 = 7$

$-6x_1 + 5x_2 + x_3 = 2$

$5x_2 + x_3 = -6$

2) $x_1 \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 5 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix}$

3) $\begin{bmatrix} 3 & -2 & 1 \\ -6 & 5 & 7 \\ 0 & 5 & 1 \end{bmatrix} \underline{x} = \begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix}$

4) $\begin{bmatrix} 1 \\ 4/7 \\ 36/7 \end{bmatrix}$

6. 1) [0 — 0 | b] in ech. form;
no [0 — 0 | b] in ech form and no free vars;
no [0 — 0 | b] in ech form and free vars

2) no 3) yes 4) no
5) 7, 4 6) yes, 4 pivots
7) no 8) no 9) yes, 4 pivots

H. 1) preserves + and scalar.

2 - 4 see week 12

I. see week 12

J. 1) $\{$ some nonzero vector orthogonal to L, some nonzero vector in L $\}$

2) $\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$; $\begin{bmatrix} \underline{v_1} & \cdots & \underline{v_n} \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} \underline{v_1} & \cdots & \underline{v_n} \end{bmatrix}^{-1}$

3) $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ or $\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$

depending on orientation