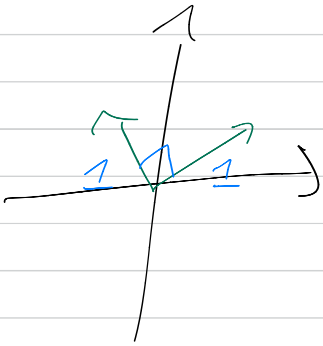
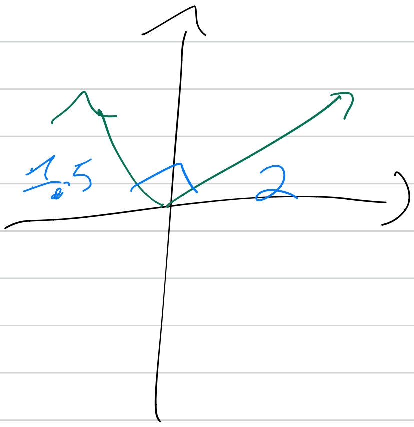
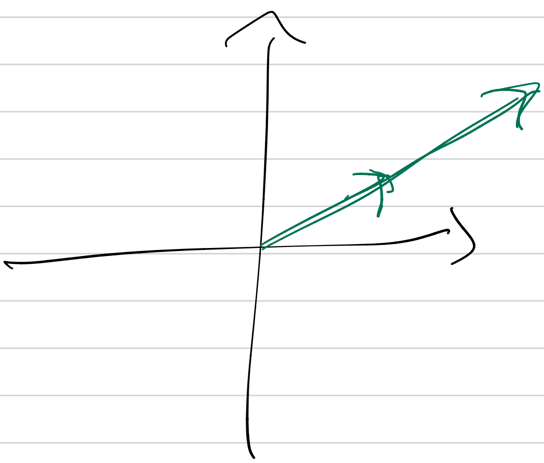
A. 1)



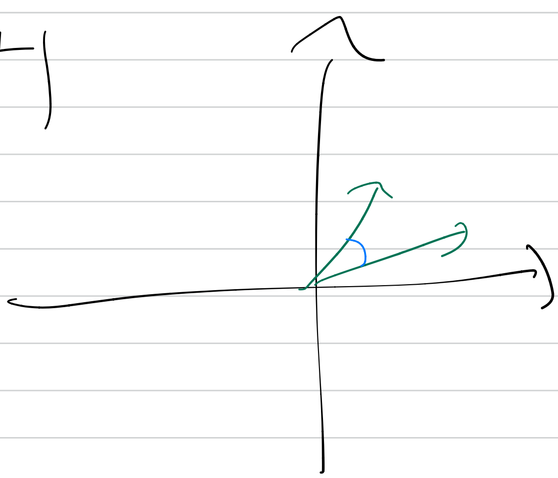
2)



3)



4)



5) not possible with nonzero vectors

(could throw in **0** vector with (1) or (2)).

B. 1) 25; 0; 0; 1; 0; 25

2) 5; 1; 5

3) yes

4) no

5) $1/5$

6) $\begin{bmatrix} 25 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

C. 1) yes: it is the nullspace of the matrix $[4 \ 1 \ 1]$.

2) $\text{Null}([4 \ 1 \ 1])$

$= \left\{ x_2 \begin{bmatrix} -1/4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1/4 \\ 0 \\ 1 \end{bmatrix} \right\}$

a basis is $\left\{ \begin{bmatrix} -1/4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

not orthogonal
not orthonormal

3) yes: $4(-\tfrac{1}{3}) + 1(\tfrac{2}{3}) + 1(\tfrac{2}{3}) = 0$

likewise for $\underline{v}$.

4) yes: $\begin{bmatrix} \underline{u} & \underline{v} \end{bmatrix}^T \begin{bmatrix} \underline{u} & \underline{v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

5) LI since ON

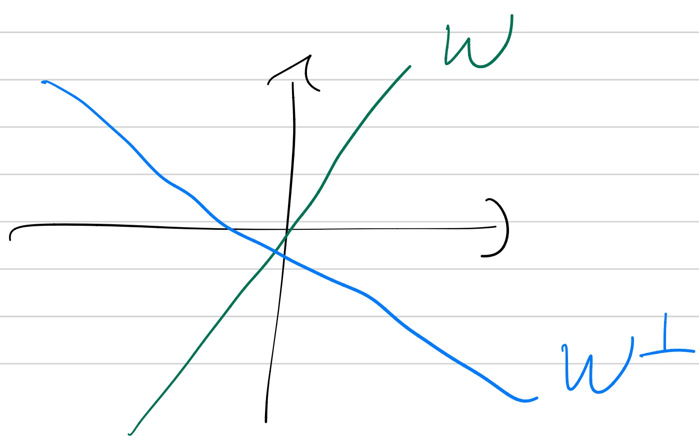
spans b/c ~~it~~ is 2 LI vectors in a 2-dimensional space.

6)

$$\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \cdot \underline{u} = 3$$

$$\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \cdot \underline{v} = -\sqrt{2} \quad \rightsquigarrow \quad \begin{bmatrix} 3 \\ -\sqrt{2} \end{bmatrix}$$

D. 1)



2) $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$

3) $\left\{ \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} \right\}$

4)

$$\left(\begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right) \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} = \begin{bmatrix} 3/5 \\ 9/5 \end{bmatrix}$$

$$\left(\begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} \cdot \begin{bmatrix} -5 \\ -5 \end{bmatrix} \right) \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$$

5) $\left\{ \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$

E. 1) It is the closest point to itself...

2) $\underline{w} \in \operatorname{Span} \mathcal{U}$

$\Rightarrow \underline{w} = c_1\underline{u_1} + \cdots + c_t\underline{u_t}$.

We know $c_i = \underline{u_i} \cdot \underline{w}$ by $\mathcal{U}$-coordinates formula.

Then $\operatorname{proj}_W(\underline{w}) = (\underline{w} \cdot \underline{u_1})\underline{u_1} + \cdots + (\underline{w} \cdot \underline{u_t})\underline{u_t}$

$= c_1\underline{u_1} + \cdots + c_t\underline{u_t} = \underline{w}$.

3). $\operatorname{proj}_W(\underline{v}) = UU^T\underline{v}$

So it is a linear transfm with standard matrix UU^T.

range is W.

F. 1) $W^\perp$: if $\underline{v} \in W^\perp$, $\underline{v} = \underset{W}{\underline{0}} + \underset{W^\perp}{\underline{v}}$

$\Rightarrow \operatorname{proj}_W(\underline{v}) = \underline{0}$. (and conversely)

2). $\underline{v} = \underset{\in W}{\underline{w}} + \underset{\in W^\perp}{\underline{w}'}$

Think about it from $W^\perp$'s perspective:

$$\underline{v} = \underset{\in (W^\perp)^\perp}{\underline{w}} + \underset{\in (W^\perp)}{\underline{w}'}$$

~~So~~ $\underline{w}' = \operatorname{proj}_{W^\perp}(\underline{v})$.

6. 1) $(\underline{a}+\underline{b})(\underline{a}+\underline{b}) = \underline{a}\cdot\underline{a} + 2\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{b}$

So $\|\underline{a}+\underline{b}\|^2 = \|\underline{a}\|^2 + \|\underline{b}\|^2$.

Both facts follow.

2) If $\underline{w} \in W$, write $\underline{w} = \hat{\underline{v}} - \underline{w}'$,

So $\underline{w}' = \hat{v} - \underline{w}$ for some $\underline{w} \in W$.

Then $\|\underline{z} + \underline{w}\| = \|(\underline{v} - \hat{\underline{v}}) + \underline{w}\|$

$= \|\underline{v} - \underline{w}'\| = \operatorname{dist}(\underline{v}, \underline{w})$

$$\geq \|\underline{z}\| = \|\underline{v} - \hat{\underline{v}}\| = \text{dist}(\underline{v}, \hat{\underline{v}}).$$

H. See ~~last~~ week.