The matrix of a linear transformation

Definition: If V and W are vector spaces, $T : V \to W$ is a linear transformation, $\mathcal{B} = \{\mathbf{b_1}, \dots, \mathbf{b_n}\}$ is a basis for V, and $\mathcal{C} = \{\mathbf{c_1}, \dots, \mathbf{c_m}\}$ is a basis for W, then there is a matrix M such that

$$M[\mathbf{v}]_{\mathcal{B}} = [T(\mathbf{v})]_{\mathcal{C}}.$$

The matrix M is given by the formula

$$M = \begin{bmatrix} [\mathbf{b_1}]_{\mathcal{C}} & \cdots & [\mathbf{b_n}]_{\mathcal{C}} \end{bmatrix}.$$

Linear transformations from V to V

As a special case of the previous construction, we can consider linear transformations from a vector space to itself, so we only choose one basis.

Definition: If V is a vector space, $T : V \to V$ is a linear transformation, and $\mathcal{B} = \{\mathbf{b_1}, \dots, \mathbf{b_n}\}$ is a basis for V, then the $\mathcal{B}$-matrix for T is the matrix $[T]_{\mathcal{B}}$ that satisfies the formula

$$\qquad = [T]_{\mathcal{B}}[\mathbf{x}]_{\mathcal{B}}.$$

The matrix $[T]_{\mathcal{B}}$ is given by the formula

$$[T]_{\mathcal{B}} = \begin{bmatrix} \qquad\qquad\qquad \end{bmatrix}.$$

Linear transformations on $\mathbb{R}^n$

Theorem 5.8: If $A = PDP^{-1}$ with D an $n \times n$ diagonal matrix, and $\mathcal{B}$ is the basis of $\mathbb{R}^n$ consisting of the columns of P, then D is the ____________________ for the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$.